Domestic Transmission Capacity Services Benchmarking Model: Testing Further Specifications

Report prepared for

Australian Competition and Consumer Commission

John Fallon, Michael Cunningham, and Tim Coelli

18 January 2016

Economic Insights Pty Ltd
10 By Street, Eden, NSW 2551, Australia, AUSTRALIA
Ph 0419 171 634
WEB www.economicinsights.com.au
ABN 52 060 723 631

CONTENTS

1 INTRODUCTION

This report is supplementary to Economic Insights (1 September 2015), providing additional econometric benchmarking analysis using a revised dataset.

1.1 Purpose and Scope

The Australian Competition and Consumer Commission (ACCC) has provided to Economic Insights (on 17 November 2015) a revised dataset, which is used in this study. The dataset contains details of individual contracts for Domestic Transmission Capacity Services (DTCS), at November 2014, supplied by virtually all of the active DTCS providers. The main changes from the data used in our September report are:

- Additional data associated with an arrangement between Optus and Vodafone Hutchison Australia (VHA) has been included. As a result there are over 2,000 additional records in total, including over 1,000 on regulated routes.
- Revisions have been made by the ACCC to ensure the data is consistently on a GST-excluded basis.

The main purposes of this report are to:

- re-estimate the three models shown in Table 5.1 of the Economic Insights (2015) final report, and the Data Analysis Australia (DAA) specification (DAA 2012), using the revised dataset and calculate average predicted prices by market segment
- examine options to deal with extreme outliers, including by:
    - removing the observations of extreme influence from the data sample used for model estimation
    - testing a method of estimating "robust regression" within the context of a random effects model
- examine options to address claims that different pricing drivers apply to services of 2 Mbps, including by:
    - excluding 2 Mbps contracts from the data sample used for model estimation on the basis that a different price-setting model might be used for those services
    - including a dummy variable in the model for 2 Mbps contracts to reflect different pricing patterns for these services
    - estimating a piecewise regression model, in which there is a kink in the price-capacity relationship at 2.5 Mbps (but no discontinuity as implied by the use of a dummy variable), and
- test stochastic frontier models and compare the results to the random effect specification used in our final report.

In most of the models tested in this report (with the exception of the Robust Regression random effects and stochastic frontier models), we use 10-fold cross validation to test out-of-sample performance. This method is explained in section 2.1.

1.2 Brief Description of Data

The expanded dataset for 2014 has 20,262 records in total, including 12,554 services on regulated routes (or 62 per cent) and 7,708 services on deregulated routes (or 38 per cent). The Optus-VHA data increases the overall size of the dataset by [redacted]

Table 1.1 shows the changes to the dataset in terms of the numbers of services on each route type (inter-capital, metropolitan, regional, and tail-end). [redacted]

Table 1.1: **Route class frequencies for expanded data**

	Contracts			Market Shares		
	Regulated	*Deregulated*	*Total*	*Regulated*	*Deregulated*	*Total*
	No.	No.	No.	%	%	%
Inter-capital	368	953	1,321	2.9	12.4	6.5
Metro	4,279	6,107	10,386	34.1	79.2	51.3
Regional	3,749	648	4,397	29.9	8.4	21.7
Tail-ends	4,158	.	4,158	33.1	.	20.5
Total	12,554	7,708	20,262	100.0	100.0	100.0

Source: Economic Insights analysis.

The service providers in the dataset are shown in Table 1.2. In the original dataset, [redacted]

Average values of monthly charges, connection charges, as well as capacity and distance, are shown in Table 1.3 for 2011 and 2014, including separate figures for regulated and deregulated routes. The average monthly charge over all routes in 2014 was $1,294 in the expanded dataset, which is lower than the average of $1,386 in the original data set. The average distance for contracts on all routes in 2014 is 124 km in the expanded dataset, which is lower than the average of 133 km in the original dataset.

Table 1.2: **Provider frequencies for expanded data**

	Contracts			Market Shares		
	Regulated	*Deregulated*	*Total*	*Regulated*	*Deregulated*	*Total*
	No.	No.	No.	%	%	%
Provider 1	[redacted]					
Provider 2						
Provider 3						
Provider 4						
Provider 5						
Provider 6						
Provider 7						
Provider 8						
Provider 9						
Total	12,554	7,708	20,262	100.0	100.0	100.0

Source: Economic Insights analysis.

Table 1.3 shows a comparison of the 2014 and 2011 data for the same contract parameters. Between 2011 and 2014 the average monthly charge on regulated routes increased by 5 per cent, while the average connection charge decreased by 57 per cent. Over the same period the average monthly charge on deregulated routes decreased by 31 per cent and the average connection charge decreased by 70 per cent. Between 2011 and 2014, the average capacity of contracts on regulated routes increased from 17 Mbps to 82 Mbps, and the average distance increased from 67 km to 103 km. Over the same period, the average capacity of contracts on deregulated routes increased from 102 Mbps to 272 Mbps, and the average distance increased from 144 km to 158 km.

Table 1.3: **Average contract parameters**

	Monthly charge ($)	*Connection chg. ($)*	*Capacity (Mbps)*	*Distance (km)*
2014				
Regulated	1,286	1,405	82	103
Deregulated	1,307	2,455	272	158
All routes	1,294	1,804	154	124
2011				
Regulated	1,219	3,243	17	67
Deregulated	1,898	8,206	102	144
All routes	1,425	6,618	43	90
% change*				
Regulated	5	-57	382	54
Deregulated	-31	-70	167	10
All routes	-9	-73	258	38

* Calculated as the overall change over three years, rather than as an annual average rate of change.
Source: Economic Insights analysis.

1.3 Organisation of the report

The remainder of this report is organised as follows. The main body of the report presents econometric models estimated with the revised dataset, and Appendix A presents the results of estimating most of the same models using the dataset without the additional Optus-VHA observations. For all of the models in this report there are tables showing the predicted average prices by market segment for regulated and deregulated routes.

- Section 2 presents the results of re-estimating the three models shown in Table 5.1 of our previous report (Economic Insights 2015), and the DAA specification, using the revised dataset.
- Section 3 presents some models which are designed to remove or reduce the effects of extreme outliers.
- Section 4 presents a series of different models which seek to account for the possibility of structural differences in the pricing of low capacity, short distance services.
- Section 5 presents estimates of a stochastic frontier model, as an alternative to the random effects specification used in all of the other models reported here.
- A summary discussion of the results is included in section 6.

2 RE-ESTIMATION OF PREVIOUS MODELS WITH REVISED DATASET

This section presents the results of re-estimating the three models that were presented in Table 5.1 of the Economic Insights final report, using the expanded dataset that includes the data relating to the Optus-VHA arrangement and with all data on a consistent basis in terms of excluding GST.

2.1 EI specifications

The specifications developed in our 1 September 2015 report were based on the translog cost function, which can be summarised as follows:

$$\ln C = \gamma_0 + \gamma_1 \ln y_1 + \gamma_2 \ln y_2 + \frac{1}{2}\gamma_{11}\left(\ln y_1\right)^2 + \frac{1}{2}\gamma_{22}\left(\ln y_2\right)^2 + \gamma_{12} \ln y_1 \ln y_2 + \sum_k \beta_k z_k + \varepsilon \tag{2.1}$$

Here, C is the monthly charge for the service (which is assumed to be based on the cost of supply in the competitive markets on deregulated routes); y_1 is capacity in Mbps; y_2 is the distance in km; z_k is one of a set of other explanatory variables; ε is a stochastic term which can be formulated differently in different models, but in the random effects model it is defined as: $\varepsilon_{it} = u_i + e_{it}$, where i represents a route number, and t represents a sequential number for observations on that route; u_i is normally distributed with a single value on each route, and e_{it} is normally distributed with a different value for each observation.[1] The Greek symbols in equation 2.1 represent parameters to be estimated. The z variables include route-class effects, provider-specific effects, and an effect for interface-type, and some models also include other variables.

Econometric Results

Table 2.1 shows the results of re-estimating the models presented in our final report using the revised data. The dependant variable is always the log monthly charge, and each model has the set of common explanatory variables stated above. Model (1a) also includes log route throughput, log ESA throughput and contract start date. In Model (1b), contract start date is excluded, and in Model (1c) the log route throughput and log ESA throughput variables are also excluded. The last of these models corresponds to the preferred model in the previous report.

The same models have also been estimated on randomly selected subsamples of the dataset, using a procedure known as 10-fold cross validation. This is a refinement of the method used in our final report. There we divided the sample into two parts, a randomly selected sub-

[1] Hereafter we also refer to ε as *ue* when this stochastic term comprises the two components *u* and *e*.

sample representing 10 per cent of the observations on deregulated routes, set aside for model validation, and the remaining observations to be used for estimation. The goodness-of-fit statistics were then reported separately for both the estimation and validation samples. After validation the models were re-estimated with the full dataset.

Table 2.1: **Model 1: Base case RE models (2014 data)**

	Model (1a)		**Model (1b)**		**Model (1c)**	
Predictor	*Coeff.*	*t-stat.*	*Coeff.*	*t-stat.*	*Coeff.*	*t-stat.*
constant	5.72782	35.84	4.63204	42.23	4.97396	61.35
log capacity	0.46833	42.22	0.46012	41.40	0.45826	41.26
log distance	0.05014	2.30	0.04450	2.02	0.07142	3.23
0.5(log capacity)^2	-0.03135	-11.28	-0.03030	-10.85	-0.02983	-10.69
0.5(log distance)^2	0.02330	3.83	0.02421	3.95	0.02477	3.97
(log capacity)(log distance)	-0.00420	-2.89	-0.00405	-2.77	-0.00576	-4.02
log route t'put	-0.02832	-5.55	-0.03068	-5.97	.	.
log ESA t'put	0.05572	6.33	0.05216	5.88	.	.
contract start date	-0.00006	-9.35	.	.	.	.
route class 2 (Metro)	0.08855	1.29	0.08669	1.25	0.14763	2.11
route class 3 (Regional)	0.24319	4.44	0.23657	4.28	0.25023	4.46
Provider #1	[redacted]					
Provider #3						
Provider #4						
Provider #5						
Provider #6						
Provider #7						
Provider #8						
Provider #9						
interface-type 3 (SDH)	0.23558	12.67	0.28574	15.96	0.28211	15.74
σ(u)	0.29731		0.30121		0.31154	
σ(e)	0.41807		0.42001		0.41947	

Source: Economic Insights estimation results.

Notes: * Squared correlation between fitted and actual dependent.

In the 10-fold cross-validation approach, random selection is used to divide the full sample into 10 (approximately) equal sub-samples, to serve as validation samples. The complement of each of these validations samples (i.e., the observations not included in that validation sample) serves as an estimation sample. The same model is estimated 10 times, once for each of the 10 estimation samples, and the goodness-of-fit statistics are obtained for that estimation sample and for the corresponding validation sample. These goodness-of-fit measures are averaged across the 10 models, to obtain representative within-sample and out-of-sample goodness-of-fit statistics.

Goodness-of-fit measures are presented in Table 2.2 for each of the three models shown in Table 2.1. The average within-sample and out-of-sample goodness-of-fit statistics obtained using the 10-fold cross-validation technique, are compared to the goodness-fit statistics associated with models estimated using the full sample (which are necessarily within-sample).

Table 2.2: **Model 1: Base case RE models (2014 data): Goodness-of-fit**

	Model (1a)		Model (1b)		Model (1c)	
Goodness-of-fit statistic	*10-fold validation (avg)*	*Full sample*	*10-fold validation (avg)*	*Full sample*	*10-fold validation (avg)*	*Full sample*
Within-sample						
obs	6,937	7,708	6,937	7,708	6,937	7,708
R^2*	0.68526	0.68472	0.68207	0.68149	0.67466	0.67423
BIC		10169.4		10247.4		10275.0
RMSE (based on ue)	0.50543	0.50597	0.50795	0.50850	0.51452	0.51504
MAE (based on ue)	0.36301	0.36364	0.36364	0.36430	0.37295	0.37337
Out-of-sample						
obs	771	0	771	0	771	0
R^2*	0.68332	.	0.68014	.	0.67294	.
RMSE (based on ue)	0.50702	.	0.50947	.	0.51584	.
MAE (based on ue)	0.36410	.	0.36474	.	0.37395	.

Notes: * Squared correlation between fitted and actual dependent.

Table 2.3: **Model 1: Base case RE models (2014 data): Statistical tests**

	Model (1a)		Model (1b)		Model (1c)	
	Stat.	*P-value**	*Stat.*	*P-value**	*Stat.*	*P-value**
Normality of residuals						
Doornik-Hansen(1)	4546.0	0.0000	4669.3	0.0000	4579.6	0.0000
IQR (% severe outliers) (2)†	2.34%		2.59%		2.64%	
Homoscedasticity						
Breusch-Pagan/Cook-Weisberg(6)	1101.5[a]	0.0000	1143.7[a]	0.0000	1239.8[a]	0.0000
Multicollinearity						
# VIF scores > 10	5/19		5/18		5/16	
Misspecification						
RESET (7)	26.8[a]	0.0000	9.7[a]	0.0000	9.9[a]	0.0000
Link test(8)	3.26[a]	0.001	1.53[a]	0.126	1.21[a]	0.225
Joint significance tests						
2nd order output terms (df = 3)	180.1	0.0000	168.5	0.0000	180.4	0.0000
Route classes (df = 2)	36.3	0.0000	33.7	0.0000	27.4	0.0000
Provider fixed effects (df = 7)	981.6	0.0000	985.7	0.0000	980.7	0.0000

Note: * Null hypothesis is rejected, as a standard procedure, in these tests, if P-value is less than 0.05. Equivalently, the reported statistic exceeds the critical value for that statistic; † Percentage of n = 7,708 observations; (1) $chi^2(2k)$ where k = 19 for 1st model, and k = 18 for 2nd model and k = 16 for 3rd model. (2) Severe outliers represent about 0.0002% of a normal distribution; (3) Studentized residual > 3; (4) Hat value > $3k/n$; (5) Cook's D > 5 × average Cook's D; (6) $chi^2(1)$; (7) Via powers of the dependent variable, F(3,n–k–3); (8) t-statistic on hat^2; (9) F(r,n–k–r), where r = number of parameters tested, and r = 3 for higher-order output terms, r = 2 for route classes, and r = 8 for provider-specific effects. (10) $chibar^2(1)$; a Approximate, based on OLS regression of $(y - \hat{u}_i)$ on the predictors.

Table 2.3 shows the results of statistical tests, which have been used to assist in comparing models. The results are broadly similar to those obtained with the original data sample. The hypotheses that the residuals are normally distributed and homoscedastic are rejected. The high proportion of severe outliers indicates that the frequency distribution of the residuals has much fatter tails than the normal distribution. Normality of the residuals is not essential for consistent estimates, and heteroscedasticity does not by itself signal bias in the estimated coefficients.

The tests of misspecification (the RESET test and the Link test) do not support model (1a). Both are considerably improved for models (1b) and (1c). For these two models the null hypothesis that the model is not misspecified is accepted by the Link test but rejected by the RESET test. We are aware that there are potential buyer-side factors that may influence prices but for which there is no available data. The RESET tests indicate there may be some omitted variables or other specification error.

The joint parameter tests indicate that the 2nd order output terms are jointly significant, as are the route-class effects (random effects) and the provider-specific fixed effects.

2.2 DAA specification

For completeness the DAA specification has been re-estimated with the updated dataset, and the results are shown in Table 2.4. Hereafter we refer to this model as "DAA 2015".[2]

Table 2.4: **Estimation results, DAA specification (2014 data)**

Predictor	*Coeff.*	*t-stat.*	*Predictor*	*Coeff.*	*t-stat.*
constant	5.81047	64.90	Route class # QOS:		
log capacity	0.32864	40.89	Metro#2		
log distance	0.09499	15.04	Metro#3		
protection	-0.11240	-7.36	Metro#4		
Route class:			Regional#2		
Metro	-0.23332	-2.81	Regional#3		
Regional	-0.08967	-1.05	Regional#4		
QOS:			QOS # log capacity:		
2			2		
3			3		
4			4		
Goodness-of-Fit					
R-sq	0.6532		RMSE	0.5305	

Source: Economic Insights estimates.

[2] The reference to 2015 is the year of estimation rather than the date of the data used. Similarly we refer to the model previously estimated by DAA in 2012 using 2011 data as "DAA 2012".

2.3 Average Price Effects per Market Segment

Table 2.5 shows comparisons of average price predictions using each variant of Model 1 against average actual charges and the charges predicted by the DAA 2012 model, which represents the prevailing regulated price formula. These statistics are all in terms of $ per month, and are on a comparable basis in terms of excluding GST.

On regulated routes, the overall average actual price is $1,286 per month, which is only about 35 per cent of the average predicted price using the DAA 2012 model (which is the regulated price formula), which is $3,627. The largest differences between the actual and regulated prices are for large capacity services of 200 Mbps or greater. The overall average price on regulated routes predicted by Model (1c) is $895, which is approximately 30 per cent below the average actual price on those routes.

In assessing the predictive capacity of the models comparisons are first made in relation to the predictions using the DAA 2012 model. As noted the predicted prices from the DAA 2012 model are the regulated prices that currently apply. However, there is scope for commercial negotiation to lead to lower actual prices than regulated prices.

The comparisons then consider the predictions of the new models and the DAA model re-estimated with 2014 data for deregulated actual prices, which were used to estimate the models. Finally the comparisons consider the predictions of the models for regulated actual prices.

The aim is to obtain a model which is the best predictor across the various capacities as a whole. This means that the focus of the comparisons of the predictions should be on identifying which model is the best predictor for the deregulated market. This follows since the models are based on the deregulated market and then used to obtain a benchmark 'competitive' price for the regulated market. If the new models provide superior predictions relative to the DAA 2012 model for the deregulated market they should be preferred to the DAA 2012 model. If the new models provide lower predictions for the regulated market than the DAA 2012 model but these predictions are not good predictors of the actual prices in the regulated market that is of little consequence because commercial negotiation can lead to actual prices in the regulated market that are less than the regulated prices.

Furthermore if actual prices in the regulated market are less than the predictions with the new models which are also less than current regulated prices there is no reason to expect that actual prices would necessarily increase if the predicted prices of the new models were the new regulated prices. In these circumstances, changing the cap[3] in this way may not necessarily lower actual prices in the regulated market but also not be likely to lead to increased actual prices in the regulated market.

Table 2.6 shows the percentage differences of actual and predicted prices relative to the DAA 2012 model (i.e. regulated prices). The predicted prices from all of the models and almost all

[3] The term "cap" is used for illustrative purposes. It is understood that parties are not required to contract at the DAA 2012 prices, as they are standing offers rather than caps. This is evident for services of less than 50 Mbps on regional routes which are priced higher than the 2012 DTCS FAD price.

actual prices are substantially lower than the predicted prices using the DAA 2012 model. The main exceptions are services of less than 50 Mbps on regional routes, where the actual prices are considerably higher than the DAA 2012 (i.e., regulated) average prices.

Table 2.7 shows the percentage differences of the predicted prices of the models estimated with 2014 data against the actual deregulated prices and actual regulated prices.

The new models on average predict prices for deregulated routes that are considerably lower than the actual deregulated prices while the DAA 2015 model predicts prices that are higher than the actual deregulated prices. Model 1(c) provides an average prediction that is 9 per cent less than actual deregulated prices while the DAA 2015 model provides an average prediction that is 12 per cent higher than actual deregulated prices. For metro routes model 1 (c) provides a prediction that matches the actual deregulated price on average whereas the DAA 2015 model predict metro prices that are 13 per cent above actual deregulated prices. Models 1(b) and 1(c) are also clearly better predictors than the DAA 2015 model for all metro services and for capacities up to 200 Mbps.

Turning to the predictions for the regulated routes, the only market segments where the average price predicted by either Model (1b) or Model (1c) is significantly greater than the average actual price are:

(i) contracts of less than 5 Mbps on Metro routes (for which they are 5 per cent higher with Model (1b) and 10 per cent higher with Model (1c)) and contracts of more than 200 Mbps on Metro routes (for which they are 1 per cent higher with Model (1b) and 7 per cent higher with Model (1c))

(ii) contracts for tail-end services, for which the Model (1b) predicted price is 17 per cent higher than the actual price, and the Model (1c) predicted price is 22 per cent higher.

In all other market segments on regulated routes the average prices predicted by Models (1b) and (1c) are either not significantly different from, or (in most cases) lower than the average actual price.

Elasticities of Cost with Respect to Each Output

When comparing model specification it will be useful to calculate the elasticities of cost with respect to each output, and the following formulas are used to do so:

(2.2) $$\frac{\partial \ln C}{\partial \ln y_1} = \gamma_1 + \gamma_{11} \ln y_1 + \gamma_{12} \ln y_2$$

(2.3) $$\frac{\partial \ln C}{\partial \ln y_2} = \gamma_2 + \gamma_{22} \ln y_2 + \gamma_{12} \ln y_1$$

When evaluating these elasticities, three alternate values of each variable is used, the 10th percentile, the mean and the 90th percentile. These elasticities are calculated separately for deregulated routes and for regulated routes.

Table 2.8 presents the elasticities of cost with respect to capacity and distance for each of the three models presented in Table 2.1.

Table 2.5: **Model 1: Average actual and predicted prices by market segment ($/month)**

Capacity class	*Route type*	*Actual*	*DAA 2012*	*DAA 2015*	*Model (1a)*	*Model (1b)*	*Models (1c)*
Regulated routes							
Capacity < 5 Mbps	Intercapital	1209	1365	944	753	844	883
	Metro	378	458	420	346	395	417
	Regional	1043	782	579	523	595	641
	All (excl. TEs)	713	635	509	440	501	535
Capacity 5 - 50 Mbps	Intercapital	2881	6192	2056	1556	1664	1690
	Metro	1074	1676	905	670	726	772
	Regional	3995	3409	1306	1216	1339	1460
	All (excl. TEs)	2385	2807	1178	971	1058	1132
Capacity 50 - 200 Mbps	Intercapital	6677	13969	3034	2359	2552	2499
	Metro	1524	6405	1708	1363	1461	1550
	Regional	6129	11946	2478	2196	2365	2519
	All (excl. TEs)	3818	9253	2122	1782	1917	2024
Capacity 200+ Mbps	Intercapital	11784	52403	5909	3906	4188	4064
	Metro	2023	13770	2557	1912	2045	2158
	Regional	5198	36426	4253	3257	3518	3683
	All (excl. TEs)	3834	24883	3416	2554	2746	2865
All capacities	Intercapital	4437	12861	2415	1776	1924	1914
	Metro	824	3106	927	727	794	840
	Regional	2472	6116	1283	1100	1209	1289
	All (excl. TEs)	1718	4877	1151	939	1029	1088
	Tail-end	[redacted]	1101	649	429	483	505
	All (incl. TEs)	[redacted]	3627	985	770	848	895
Deregulated							
Capacity < 5 Mbps	Intercapital	812	1340	938	706	787	818
	Metro	398	440	425	340	387	406
	Regional	504	653	549	466	531	557
	All (excl. TEs)	434	516	469	375	426	447
Capacity 5 - 50 Mbps	Intercapital	1534	4853	1762	1335	1431	1452
	Metro	859	1489	814	637	693	720
	Regional	1884	4294	1432	1262	1362	1483
	All (excl. TEs)	1010	2161	997	778	842	872
Capacity 50 - 200 Mbps	Intercapital	2432	13215	2982	2314	2481	2470
	Metro	1399	5202	1550	1243	1335	1395
	Regional	3475	10279	2341	2018	2172	2302
	All (excl. TEs)	1714	6723	1812	1454	1562	1618
Capacity 200+ Mbps	Intercapital	7621	120388	9415	5235	5655	5422
	Metro	1979	14698	2586	1920	2058	2140
	Regional	4449	32605	4069	3160	3422	3563
	All (excl. TEs)	3405	38549	4160	2737	2946	2968
All capacities	Intercapital	3290	38367	4008	2516	2716	2661
	Metro	923	3673	1043	811	882	920
	Regional	2009	9274	1688	1385	1510	1588

Capacity class	*Route type*	*Actual*	*DAA 2012*	*DAA 2015*	*Model (1a)*	*Model (1b)*	*Models (1c)*
	All (excl. TEs)	1307	8433	1464	1070	1161	1191
	Tail-end	.	.	.	.	.	.
	All (incl. TEs)	1307	8433	1464	1070	1161	1191
Regulated & Deregulated							
Capacity < 5 Mbps	Intercapital	979	1350	940	726	811	845
	Metro	388	449	423	343	391	411
	Regional	980	767	576	516	588	631
	All (excl. TEs)	607	590	494	415	473	501
Capacity 5 - 50 Mbps	Intercapital	1842	5160	1829	1385	1484	1507
	Metro	911	1535	836	645	701	733
	Regional	3667	3546	1326	1223	1343	1464
	All (excl. TEs)	1465	2374	1057	842	913	958
Capacity 50 - 200 Mbps	Intercapital	3689	13438	2998	2327	2502	2479
	Metro	1445	5642	1608	1287	1381	1452
	Regional	5645	11642	2453	2163	2330	2480
	All (excl. TEs)	2700	7909	1957	1608	1728	1808
Capacity 200+ Mbps	Intercapital	8311	109128	8834	5015	5412	5197
	Metro	1995	14363	2575	1917	2053	2146
	Regional	4998	35407	4204	3231	3492	3651
	All (excl. TEs)	3583	32878	3851	2661	2863	2925
All capacities	Intercapital	3610	31261	3564	2310	2495	2453
	Metro	882	3439	995	776	846	887
	Regional	2404	6581	1343	1142	1254	1333
	All (excl. TEs)	1521	6579	1301	1002	1092	1137
	Tail-end	[redacted]	1101	649	429	483	505
	All (incl. TEs)	[redacted]	5455	1167	884	967	1008

Table 2.6: **Model 1: Difference in actual & predicted prices compared to DAA 2012 (%)**

Capacity class	*Route type*	*Actual*	*DAA 2015*	*Model (1a)*	*Model (1b)*	*Models (1c)*
Regulated routes						
Capacity < 5 Mbps	Intercapital	-11	-31	-45	-38	-35
	Metro	-18	-8	-24	-14	-9
	Regional	33	-26	-33	-24	-18
	All (excl. TEs)	12	-20	-31	-21	-16
Capacity 5 - 50 Mbps	Intercapital	-53	-67	-75	-73	-73
	Metro	-36	-46	-60	-57	-54
	Regional	17	-62	-64	-61	-57
	All (excl. TEs)	-15	-58	-65	-62	-60
Capacity 50 - 200 Mbps	Intercapital	-52	-78	-83	-82	-82
	Metro	-76	-73	-79	-77	-76
	Regional	-49	-79	-82	-80	-79
	All (excl. TEs)	-59	-77	-81	-79	-78
Capacity 200+ Mbps	Intercapital	-78	-89	-93	-92	-92
	Metro	-85	-81	-86	-85	-84
	Regional	-86	-88	-91	-90	-90
	All (excl. TEs)	-85	-86	-90	-89	-88

Capacity class	*Route type*	*Actual*	*DAA 2015*	*Model (1a)*	*Model (1b)*	*Models (1c)*
All capacities	Intercapital	-65	-81	-86	-85	-85
	Metro	-73	-70	-77	-74	-73
	Regional	-60	-79	-82	-80	-79
	All (excl. TEs)	-65	-76	-81	-79	-78
	Tail-end	-62	-41	-61	-56	-54
	All (incl. TEs)	-65	-73	-79	-77	-75
Deregulated						
Capacity < 5 Mbps	Intercapital	-39	-30	-47	-41	-39
	Metro	-9	-3	-23	-12	-8
	Regional	-23	-16	-29	-19	-15
	All (excl. TEs)	-16	-9	-27	-18	-13
Capacity 5 - 50 Mbps	Intercapital	-68	-64	-73	-71	-70
	Metro	-42	-45	-57	-53	-52
	Regional	-56	-67	-71	-68	-65
	All (excl. TEs)	-53	-54	-64	-61	-60
Capacity 50 - 200 Mbps	Intercapital	-82	-77	-82	-81	-81
	Metro	-73	-70	-76	-74	-73
	Regional	-66	-77	-80	-79	-78
	All (excl. TEs)	-75	-73	-78	-77	-76
Capacity 200+ Mbps	Intercapital	-94	-92	-96	-95	-95
	Metro	-87	-82	-87	-86	-85
	Regional	-86	-88	-90	-90	-89
	All (excl. TEs)	-91	-89	-93	-92	-92
All capacities	Intercapital	-91	-90	-93	-93	-93
	Metro	-75	-72	-78	-76	-75
	Regional	-78	-82	-85	-84	-83
	All (excl. TEs)	-85	-83	-87	-86	-86
	Tail-end	.	.	.	.	.
	All (incl. TEs)	-85	-83	-87	-86	-86
Regulated & Deregulated						
Capacity < 5 Mbps	Intercapital	-28	-30	-46	-40	-37
	Metro	-13	-6	-24	-13	-8
	Regional	28	-25	-33	-23	-18
	All (excl. TEs)	3	-16	-30	-20	-15
Capacity 5 - 50 Mbps	Intercapital	-64	-65	-73	-71	-71
	Metro	-41	-46	-58	-54	-52
	Regional	3	-63	-66	-62	-59
	All (excl. TEs)	-38	-55	-65	-62	-60
Capacity 50 - 200 Mbps	Intercapital	-73	-78	-83	-81	-82
	Metro	-74	-72	-77	-76	-74
	Regional	-52	-79	-81	-80	-79
	All (excl. TEs)	-66	-75	-80	-78	-77
Capacity 200+ Mbps	Intercapital	-92	-92	-95	-95	-95
	Metro	-86	-82	-87	-86	-85
	Regional	-86	-88	-91	-90	-90
	All (excl. TEs)	-89	-88	-92	-91	-91
All capacities	Intercapital	-88	-89	-93	-92	-92
	Metro	-74	-71	-77	-75	-74

Capacity class	*Route type*	*Actual*	*DAA 2015*	*Model (1a)*	*Model (1b)*	*Models (1c)*
	Regional	-63	-80	-83	-81	-80
	All (excl. TEs)	-77	-80	-85	-83	-83
	Tail-end	-62	-41	-61	-56	-54
	All (incl. TEs)	-76	-79	-84	-82	-82

Table 2.7: **Model 1: Difference in predicted prices compared to Actual (%)**

Capacity class	*Route type*	*DAA 2015*	*Model (1a)*	*Model (1b)*	*Models (1c)*
Regulated routes					
Capacity < 5 Mbps	Intercapital	-22	-38	-30	-27
	Metro	11	-8	5	10
	Regional	-44	-50	-43	-39
	All (excl. TEs)	-29	-38	-30	-25
Capacity 5 - 50 Mbps	Intercapital	-29	-46	-42	-41
	Metro	-16	-38	-32	-28
	Regional	-67	-70	-66	-63
	All (excl. TEs)	-51	-59	-56	-53
Capacity 50 - 200 Mbps	Intercapital	-55	-65	-62	-63
	Metro	12	-11	-4	2
	Regional	-60	-64	-61	-59
	All (excl. TEs)	-44	-53	-50	-47
Capacity 200+ Mbps	Intercapital	-50	-67	-64	-66
	Metro	26	-5	1	7
	Regional	-18	-37	-32	-29
	All (excl. TEs)	-11	-33	-28	-25
All capacities	Intercapital	-46	-60	-57	-57
	Metro	13	-12	-4	2
	Regional	-48	-56	-51	-48
	All (excl. TEs)	-33	-45	-40	-37
	Tail-end	57	4	17	22
	All (incl. TEs)	-23	-40	-34	-30
Deregulated					
Capacity < 5 Mbps	Intercapital	15	-13	-3	1
	Metro	7	-15	-3	2
	Regional	9	-8	5	10
	All (excl. TEs)	8	-14	-2	3
Capacity 5 - 50 Mbps	Intercapital	15	-13	-7	-5
	Metro	-5	-26	-19	-16
	Regional	-24	-33	-28	-21
	All (excl. TEs)	-1	-23	-17	-14
Capacity 50 - 200 Mbps	Intercapital	23	-5	2	2
	Metro	11	-11	-5	0
	Regional	-33	-42	-38	-34

Capacity class	*Route type*	*DAA 2015*	*Model (1a)*	*Model (1b)*	*Models (1c)*
	All (excl. TEs)	6	-15	-9	-6
Capacity 200+ Mbps	Intercapital	24	-31	-26	-29
	Metro	31	-3	4	8
	Regional	-9	-29	-23	-20
	All (excl. TEs)	22	-20	-13	-13
All capacities	Intercapital	22	-24	-17	-19
	Metro	13	-12	-4	0
	Regional	-16	-31	-25	-21
	All (excl. TEs)	12	-18	-11	-9
	Tail-end	.	.	.	.
	All (incl. TEs)	12	-18	-11	-9
Regulated & Deregulated					
Capacity < 5 Mbps	Intercapital	-4	-26	-17	-14
	Metro	9	-12	1	6
	Regional	-41	-47	-40	-36
	All (excl. TEs)	-19	-32	-22	-17
Capacity 5 - 50 Mbps	Intercapital	-1	-25	-19	-18
	Metro	-8	-29	-23	-20
	Regional	-64	-67	-63	-60
	All (excl. TEs)	-28	-43	-38	-35
Capacity 50 - 200 Mbps	Intercapital	-19	-37	-32	-33
	Metro	11	-11	-4	0
	Regional	-57	-62	-59	-56
	All (excl. TEs)	-28	-40	-36	-33
Capacity 200+ Mbps	Intercapital	6	-40	-35	-37
	Metro	29	-4	3	8
	Regional	-16	-35	-30	-27
	All (excl. TEs)	7	-26	-20	-18
All capacities	Intercapital	-1	-36	-31	-32
	Metro	13	-12	-4	1
	Regional	-44	-53	-48	-45
	All (excl. TEs)	-14	-34	-28	-25
	Tail-end	57	4	17	22
	All (incl. TEs)	-10	-32	-25	-22

Table 2.8: **Model 1: Base case RE models (2014 data): Cost elasticities**

	Model (1a)		**Model (1b)**		**Model (1c)**	
	capacity	*distance*	*capacity*	*Distance*	*capacity*	*distance*
Deregulated routes						
10th	0.444	0.060	0.437	0.055	0.434	0.081
50th	0.368	0.097	0.363	0.094	0.358	0.118
90th	0.262	0.179	0.261	0.180	0.250	0.201
Regulated routes						
10th	0.441	0.080	0.433	0.075	0.430	0.102
50th	0.398	0.119	0.392	0.116	0.386	0.142
90th	0.285	0.168	0.283	0.168	0.274	0.190

Source: Economic Insights.

3 SENSITIVITY OF RESULTS TO OUTLIERS

In its submission to the ACCC's Draft Report, CEG (2015) stated that the random effects models presented in the Economic Insights final report were not robust to outliers. This section considers two quite different approaches suggested by CEG, namely:

- to remove the most extreme outliers
- to utilise "robust regression" techniques to estimate the random effects model.

3.1 Excluding Extreme Observations

One approach that can be tested is to simply exclude all of the most severe outliers. In our final report, we showed that for models equivalent in form to Models (1b) and (1c), the number of observations with Cook's Distance greater than 5 times the mean represented approximately 3.4 per cent of the total sample (Economic Insights 2015 p.95).[4] CEG suggest removing all of these observations and testing the result. This section considers the effect of removing the most extreme observations that have greatest influence on the estimated model.

We identified 329 observations that had greatest influence on Model 1, using the following method:

- For each of Models (1a), (1b) and (1c), as reported in Table 2.1, the observations with greatest influence were identified being those with Cook's Distance > $4 / (n - k - 1)$, where n is the number of observations, and k is the number of explanatory variables in the model.
- Only those observations that met this criterion for *all three* models were chosen as the observations with greatest influence.

While removing the most influential observations does demonstrate the effect of those observations on the overall results, the validity of excluding them depends entirely on whether the observations are correctly measured or are data errors. The appropriate response to highly influential observations depends on whether they are believed to reflect the data generation process or are inconsistent with it. As Greene points out:

> *since the distribution of disturbances would anticipate a certain small percentage of extreme observations in any event, simply singling out observations with large residuals is actually a dubious exercise. On the other hand, one might suspect that the outlying observations are actually generated from a different population.* (Greene 2012 p.141)

If the extreme observations are actually correct measurements, then we may be removing the most important observations in the sample. Another risk of removing outliers is that their identification is model-specific. Thus, the identification of the observations with largest Cook's Distances is based on a first-round model, the estimated parameters of which are

[4] These are the observations with highest influence because they are outliers with high leverage. The degree to which an observation is an outlier or has high leverage is model-specific.

influenced by those and other outlying observations. The second-round process of removing the selected outliers depends on an underlying *assumption* that this procedure will converge towards the 'true' underlying model, rather than diverge further away from it, which is also quite possible.

Our view is that outliers should not be removed unless it seems likely that the outlier reflects a data error or does not contain relevant information. And for this reason, data providers were asked to recheck the accuracy of many of the more extreme outliers. The general preference to retain outliers, if they are accurate observations, is because influential observations may actually be the most valuable observations in the data set. If outliers are a problem, it may be desirable to test alternative estimation methods that give them less weight (e.g., a minimum absolute errors estimator), which we did in our final report with the quantile regression estimates.

Despite this methodological preference, we have carried out a sensitivity analysis of the effect of excluding the most influential observations. However, unlike the CEG submission, our principal focus is on the effect that such exclusions have on the model's out-of-sample predictions, including the effects on predicted prices on regulated routes, rather than on *the effect of excluding outliers on the within-sample goodness of fit as emphasised by CEG*. This distinction is vitally important. CEG states that:

> *In terms of the fit of the model, the adjusted R squares for the random effects model would increase by 4.37% after the exclusion of outliers. It is arguable that proper treatment of outliers could significantly improve the fit of the model.* (CEG 2015, p.11)

It is true that if the most extreme outliers are removed, the within-sample fit of the model must improve, and the adjusted R^2 is a measure of within-sample goodness-of-fit. Indeed, if the most influential observations from this second-round regression were in turn removed, and the model re-estimated a third time, the fit of the model within the reduced sample would be further improved, and so on. Needless to say, we do not consider this to be a valid method of econometric estimation.

Econometric Results

Table 3.1 shows the results of re-estimating the three specifications that were presented in Table 2.1 while excluding the 329 observations of greatest influence, identified from Model 1 by the method previously explained.

Table 3.2 shows the goodness-of-fit statistics. It is important to note that when carrying out the 10-fold cross-validation exercise, the following method has been used. The same set of validation and estimation samples is selected as for Model 1,[5] and then the highly influential observations are excluded at the estimation stage from each estimation sample. The within-sample goodness-of-fit statistics are based on the estimation sample and exclude the highly influential observations. On the other hand, the highly influential observations that belong to

[5] That is, they are randomly selected using the same seed.

the 10 validation samples are retained in those validation samples for the purpose of calculating out-of-sample goodness-of-fit. This means that the out-of-sample goodness-of-fit statistics are based on validation samples that include outliers, and are directly comparable to those in Table 2.2 relating to Model 1.

Almost all of the statistics shown in Table 3.2 for the out-of-sample goodness-of-fit are either slightly or significantly inferior to those for Model 1 shown in Table 2.2. This finding does not lend support to the method of removing the most extreme observations from the estimation sample.

Table 3.1: **Model 2: RE models excl. influential obs. (2014 data)**

	Model (2a)		**Model (2b)**		**Model (2c)**	
Predictor	*Coeff.*	*t-stat.*	*Coeff.*	*t-stat.*	*Coeff.*	*t-stat.*
constant	5.31699	35.92	4.63883	41.03	4.92757	58.75
log capacity	0.44435	45.51	0.43798	44.94	0.43718	44.88
log distance	0.06770	2.87	0.06413	2.70	0.08923	3.75
0.5(log capacity)^2	-0.02595	-10.32	-0.02494	-9.91	-0.02460	-9.78
0.5(log distance)^2	0.02108	3.31	0.02167	3.38	0.02176	3.35
(log capacity)(log distance)	-0.00330	-2.47	-0.00321	-2.39	-0.00482	-3.68
log route t'put	-0.02816	-5.40	-0.02973	-5.66	.	.
log ESA t'put	0.04809	5.30	0.04560	4.99	.	.
contract start date	-0.00004	-7.02	.	.	.	.
route class 2 (Metro)	0.10795	1.53	0.10734	1.51	0.15708	2.21
route class 3 (Regional)	0.24692	4.48	0.24413	4.40	0.25346	4.51
Provider #1						
Provider #3						
Provider #4						
Provider #5						
Provider #6						
Provider #7						
Provider #8						
Provider #9						
interface-type 3 (SDH)	0.27403	17.10	0.30453	19.69	0.30162	19.49
σ(u)	0.36469		0.36854		0.37538	
σ(e)	0.32218		0.32279		0.32256	

Source: Economic Insights estimation results.

Notes: * Squared correlation between fitted and actual dependent.

Table 3.3 shows the results of statistical tests, which have been used to assist in comparing models. Aside from the obvious fact that there are now fewer severe outliers, the test results are broadly similar to those obtained with the original data sample, which have been discussed above.

Table 3.2: **Model 2: RE models excl. influential obs. (2014 data): Goodness-of-fit**

Goodness-of-fit statistic	**Model (2a)** *10-fold validation (avg)*	*Full sample*	**Model (2b)** *10-fold validation (avg)*	*Full sample*	**Model (2c)** *10-fold validation (avg)*	*Full sample*
Within-sample						
obs	6,641	7,379	6,641	7,379	6,641	7,379
R^2*	0.71828	0.71762	0.71630	0.71562	0.70954	0.70913
BIC		6944.4		6984.7		7004.5
RMSE (based on ue)	0.45327	0.45393	0.45486	0.45552	0.46102	0.46155
MAE (based on ue)	0.33385	0.33467	0.33459	0.33541	0.34303	0.34352
Out-of-sample						
obs	771	0	771	0	771	0
R^2*	0.67839	.	0.67554	.	0.66886	.
RMSE (based on ue)	0.51145	.	0.51372	.	0.51981	.
MAE (based on ue)	0.36569	.	0.36673	.	0.37486	.

Notes: * Squared correlation between fitted and actual dependent.

Table 3.3: **Model 2: RE models excl. influential obs. (2014 data): Statistical tests**

	Model (2a) *Stat.*	*P-value**	**Model (2b)** *Stat.*	*P-value**	**Model (2c)** *Stat.*	*P-value**
Normality of residuals						
Doornik-Hansen(1)	1126.2	0.0000	1089.7	0.0000	1101.3	0.0000
IQR (% severe outliers) (2)†	1.15%		1.15%		1.18%	
Homoscedasticity						
Breusch-Pagan/Cook-Weisberg(6)	59.3[a]	0.0000	65.0[a]	0.0000	75.6[a]	0.0000
Multicollinearity						
# VIF scores > 10	5/19		5/18		5/16	
Misspecification						
RESET (7)	22.3[a]	0.0000	12.9[a]	0.0000	12.1[a]	0.0000
Link test(8)	2.96[a]	0.003	2.15[a]	0.032	1.93[a]	0.054
Joint significance tests						
2nd order output terms (df = 3)	145.3	0.0000	135.5	0.0000	146.9	0.0000
Route classes (df = 2)	32.6	0.0000	31.3	0.0000	26.6	0.0000
Provider fixed effects (df = 7)	1226.5	0.0000	1238.2	0.0000	1228.9	0.0000

Note: * Null hypothesis is rejected, as a standard procedure, in these tests, if P-value is less than 0.05. Equivalently, the reported statistic exceeds the critical value for that statistic; † Percentage of n = 7,379 observations; (1) $chi^2(2k)$ where k = 19 for 1st model, and k = 18 for 2nd model and k = 16 for 3rd model. (2) Severe outliers represent about 0.0002% of a normal distribution; (3) Studentized residual > 3; (4) Hat value > $3k/n$; (5) Cook's D > 5 × average Cook's D; (6) $chi^2(1)$; (7) Via powers of the dependent variable, $F(3,n–k–3)$; (8) t-statistic on hat^2; (9) $F(r,n–k–r)$, where r = number of parameters tested, and r = 3 for higher-order output terms, r = 2 for route classes, and r = 8 for provider-specific effects. (10) $chibar^2(1)$; a Approximate, based on OLS regression of $(y - \hat{u}_i)$ on the predictors.

The elasticities of cost with respect to capacity and distance are shown in Table 3.4. They are all positive and the magnitudes of these elasticities are acceptable.

Table 3.4: **Model 2: (2014 data): Cost elasticities**

	Model (2a)		Model (2b)		Model (2c)	
	capacity	*distance*	*capacity*	*distance*	*capacity*	*distance*
Deregulated routes						
10th	0.425	0.077	0.419	0.074	0.418	0.098
50th	0.362	0.112	0.359	0.110	0.355	0.131
90th	0.275	0.187	0.275	0.188	0.265	0.205
Regulated routes						
10th	0.422	0.095	0.416	0.092	0.413	0.116
50th	0.387	0.130	0.382	0.129	0.377	0.152
90th	0.294	0.176	0.293	0.177	0.284	0.194

Source: Economic Insights.

Predicted Average Prices per Market Segment

Table 3.5 shows actual charges, the average price predictions using each of the three variants of Model 2, and charges predicted by the DAA 2012 model (the prevailing regulated price formula). Table 3.6 provides a comparison by showing the percentage differences of the Model 2 price predictions against the DAA 2012 (or regulated prices). Table 3.7 shows the percentage differences of Model 2 predictions and DAA 2012 predictions against actual regulated and deregulated prices.

The overall average predicted price on regulated routes is $835 per month when using Model (2b) and $876 per month when using Model (2c), which are slightly lower than the corresponding predicted prices of $848 and $895 using Models (1b) and (1c) respectively (Tables 3.5 and 2.5). Thus removing the extreme observations when estimating the models results in average predicted prices on regulated routes being about 2 per cent lower than was the case before removing the outliers. For specification (2a), the average predicted price is slightly increased compared to (1a).

The predicted prices from all of the model 2 specifications are substantially lower than the predicted prices using the DAA 2012 model for both regulated and deregulated market segments (Table 3.6).

The model 2 specifications on average predict prices for deregulated routes that are lower than the actual deregulated prices (11 to 17 per cent) while the DAA 2012 model exhibits very poor predictive capacity, particularly for higher capacity services (Table 3.7).

Turning to the predictions for the regulated routes:

- The price predictions of Model (2b) for contracts of less than 5 Mbps on regulated Metro routes are 15 per cent lower than DAA 2012 (i.e., current regulated) prices, which are the current regulated prices (Table 3.6). This compares to the predictions of Model (1b), which are on average 14 per cent lower than DAA 2012 (Table 2.6). They are 3 per cent higher than actual prices for regulated routes in this market segment (Table 3.7). This

compares to the prediction of Model (1b), which were on average 5 per cent higher than actual prices (Table 2.7).

- The price predictions of Model (2c) for contracts of less than 5 Mbps on regulated Metro routes are 10 per cent lower than current regulated prices (Table 3.6). This can be compared to the predictions of Model (1c), which were on average 9 per cent lower than DAA 2012 (Table 2.6). They are 9 per cent higher than actual prices for these regulated routes (Table 3.7). This compares to the predictions of Model (1c), which were on average 10 per cent higher (Table 2.7).

This comparison suggests that removing outliers does not substantially change the relativities between predicted prices and current regulated prices for Metro services less than 5 Mbps. It also does not have a significant effect in solving the issue emphasised by Optus concerning predicted prices for low capacity Metro contracts being higher than prevailing actual prices.

Although the EI models predict lower prices in every market segment when compared to the DAA 2012 model prices, which are the current regulated prices, Models (2c) and (1c) predict prices that exceed actual prices for contracts of more than 200 Mbps on Metro regulated routes. The difference of 4 per cent with Model (2c) is lower than the corresponding difference of 7 per cent with Model (1c). Predicted prices from both Models (2b) and (1b) are 85 per cent lower than current regulated prices for contracts of more than 200 Mbps on Metro regulated routes.

With regard to tail end services, the average predicted prices of Models (2b) and (2c) are 58 per cent and 56 per cent lower than current regulated prices respectively. When compared against actual prices, Model (2c) predictions are 17 per cent higher, and with Model (2b) the difference is 12 per cent.

In summary, the findings indicate that, after removing the observations of greatest influence:

- the out-of-sample goodness-of-fit of the models is reduced;
- the relativities between predicted prices and current regulated prices for metro services less than 5 Mbps, which were of most concern to Optus, are not significantly different to those obtained without removing those observations. Also, Optus' concern that predicted prices for low capacity services on regulated metro routes were higher than actual prices is not materially ameliorated.

These findings suggest that there does not appear to be any particular benefits to removing the most highly influential observations, and we would suggest they not be excluded from the data sample used for estimation.

Table 3.5: **Model 2: Average actual and predicted prices by market segment ($/month)**

Capacity class	*Route type*	*Actual*	*DAA 2012*	*Model (2a)*	*Model (2b)*	*Models (2c)*
Regulated routes						
Capacity < 5 Mbps	Intercapital	1209	1365	785	843	879
	Metro	378	458	359	390	410
	Regional	1043	782	551	598	638
	All (excl. TEs)	713	635	461	500	530
Capacity 5 - 50 Mbps	Intercapital	2881	6192	1554	1619	1645
	Metro	1074	1676	665	699	740
	Regional	3995	3409	1250	1329	1432
	All (excl. TEs)	2385	2807	981	1036	1100
Capacity 50 - 200 Mbps	Intercapital	6677	13969	2388	2505	2468
	Metro	1524	6405	1351	1411	1491
	Regional	6129	11946	2231	2339	2471
	All (excl. TEs)	3818	9253	1793	1877	1971
Capacity 200+ Mbps	Intercapital	11784	52403	4048	4231	4128
	Metro	2023	13770	1914	1998	2102
	Regional	5198	36426	3398	3575	3719
	All (excl. TEs)	3834	24883	2619	2745	2852
All capacities	Intercapital	4437	12861	1816	1908	1904
	Metro	824	3106	733	775	817
	Regional	2472	6116	1140	1211	1281
	All (excl. TEs)	1718	4877	962	1019	1072
	Tail-end	[redacted]	1101	429	462	482
	All (incl. TEs)	[redacted]	3627	785	835	876
Deregulated						
Capacity < 5 Mbps	Intercapital	812	1340	733	784	813
	Metro	398	440	350	380	398
	Regional	504	653	486	528	551
	All (excl. TEs)	434	516	387	420	439
Capacity 5 - 50 Mbps	Intercapital	1534	4853	1334	1392	1414
	Metro	859	1489	622	655	680
	Regional	1884	4294	1277	1341	1443
	All (excl. TEs)	1010	2161	766	805	832
Capacity 50 - 200 Mbps	Intercapital	2432	13215	2330	2431	2428
	Metro	1399	5202	1216	1271	1326
	Regional	3475	10279	2040	2138	2250
	All (excl. TEs)	1714	6723	1437	1502	1554
Capacity 200+ Mbps	Intercapital	7621	120388	5634	5929	5718
	Metro	1979	14698	1904	1990	2068
	Regional	4449	32605	3283	3459	3583
	All (excl. TEs)	3405	38549	2821	2960	2982
All capacities	Intercapital	3290	38367	2637	2769	2722
	Metro	923	3673	805	848	884
	Regional	2009	9274	1425	1507	1575
	All (excl. TEs)	1307	8433	1083	1141	1169
	Tail-end	.	.	.	.	.
	All (incl. TEs)	1307	8433	1083	1141	1169

Capacity class	*Route type*	*Actual*	*DAA 2012*	*Model (2a)*	*Model (2b)*	*Models (2c)*
Regulated & Deregulated						
Capacity < 5 Mbps	Intercapital	979	1350	754	808	841
	Metro	388	449	355	385	404
	Regional	980	767	543	590	627
	All (excl. TEs)	607	590	433	470	495
Capacity 5 - 50 Mbps	Intercapital	1842	5160	1385	1444	1467
	Metro	911	1535	632	666	695
	Regional	3667	3546	1254	1330	1434
	All (excl. TEs)	1465	2374	837	881	921
Capacity 50 - 200 Mbps	Intercapital	3689	13438	2347	2453	2440
	Metro	1445	5642	1265	1322	1386
	Regional	5645	11642	2196	2303	2431
	All (excl. TEs)	2700	7909	1604	1678	1750
Capacity 200+ Mbps	Intercapital	8311	109128	5371	5648	5455
	Metro	1995	14363	1908	1993	2081
	Regional	4998	35407	3367	3544	3683
	All (excl. TEs)	3583	32878	2737	2871	2928
All capacities	Intercapital	3610	31261	2408	2529	2494
	Metro	882	3439	775	818	856
	Regional	2404	6581	1182	1255	1324
	All (excl. TEs)	1521	6579	1020	1078	1118
	Tail-end	[redacted]	1101	429	462	482
	All (incl. TEs)	[redacted]	5455	899	951	988

Table 3.6: **Model 2: Difference in predicted prices compared to DAA 2012 (%)**

Capacity class	*Route type*	*Model (2a)*	*Model (2b)*	*Models (2c)*
Regulated routes				
Capacity < 5 Mbps	Intercapital	-43	-38	-36
	Metro	-22	-15	-10
	Regional	-30	-24	-18
	All (excl. TEs)	-27	-21	-17
Capacity 5 - 50 Mbps	Intercapital	-75	-74	-73
	Metro	-60	-58	-56
	Regional	-63	-61	-58
	All (excl. TEs)	-65	-63	-61
Capacity 50 - 200 Mbps	Intercapital	-83	-82	-82
	Metro	-79	-78	-77
	Regional	-81	-80	-79
	All (excl. TEs)	-81	-80	-79
Capacity 200+ Mbps	Intercapital	-92	-92	-92
	Metro	-86	-85	-85
	Regional	-91	-90	-90
	All (excl. TEs)	-89	-89	-89
All capacities	Intercapital	-86	-85	-85
	Metro	-76	-75	-74
	Regional	-81	-80	-79
	All (excl. TEs)	-80	-79	-78

Capacity class	*Route type*	*Model (2a)*	*Model (2b)*	*Models (2c)*
	Tail-end	-61	-58	-56
	All (incl. TEs)	-78	-77	-76
Deregulated				
Capacity < 5 Mbps	Intercapital	-45	-42	-39
	Metro	-20	-14	-10
	Regional	-26	-19	-16
	All (excl. TEs)	-25	-19	-15
Capacity 5 - 50 Mbps	Intercapital	-73	-71	-71
	Metro	-58	-56	-54
	Regional	-70	-69	-66
	All (excl. TEs)	-65	-63	-61
Capacity 50 - 200 Mbps	Intercapital	-82	-82	-82
	Metro	-77	-76	-75
	Regional	-80	-79	-78
	All (excl. TEs)	-79	-78	-77
Capacity 200+ Mbps	Intercapital	-95	-95	-95
	Metro	-87	-86	-86
	Regional	-90	-89	-89
	All (excl. TEs)	-93	-92	-92
All capacities	Intercapital	-93	-93	-93
	Metro	-78	-77	-76
	Regional	-85	-84	-83
	All (excl. TEs)	-87	-86	-86
	Tail-end	.	.	.
	All (incl. TEs)	-87	-86	-86
Regulated & Deregulated				
Capacity < 5 Mbps	Intercapital	-44	-40	-38
	Metro	-21	-14	-10
	Regional	-29	-23	-18
	All (excl. TEs)	-27	-20	-16
Capacity 5 - 50 Mbps	Intercapital	-73	-72	-72
	Metro	-59	-57	-55
	Regional	-65	-62	-60
	All (excl. TEs)	-65	-63	-61
Capacity 50 - 200 Mbps	Intercapital	-83	-82	-82
	Metro	-78	-77	-75
	Regional	-81	-80	-79
	All (excl. TEs)	-80	-79	-78
Capacity 200+ Mbps	Intercapital	-95	-95	-95
	Metro	-87	-86	-86
	Regional	-90	-90	-90
	All (excl. TEs)	-92	-91	-91
All capacities	Intercapital	-92	-92	-92
	Metro	-77	-76	-75
	Regional	-82	-81	-80
	All (excl. TEs)	-84	-84	-83
	Tail-end	-61	-58	-56
	All (incl. TEs)	-84	-83	-82

Table 3.7: **Model 2: Difference in predicted prices compared to actual (%)**

Capacity class	*Route type*	*DAA 2012*	*Model (2a)*	*Model (2b)*	*Models (2c)*
Regulated routes					
Capacity < 5 Mbps	Intercapital	13	-35	-30	-27
	Metro	21	-5	3	9
	Regional	-25	-47	-43	-39
	All (excl. TEs)	-11	-35	-30	-26
Capacity 5 - 50 Mbps	Intercapital	115	-46	-44	-43
	Metro	56	-38	-35	-31
	Regional	-15	-69	-67	-64
	All (excl. TEs)	18	-59	-57	-54
Capacity 50 - 200 Mbps	Intercapital	109	-64	-62	-63
	Metro	320	-11	-7	-2
	Regional	95	-64	-62	-60
	All (excl. TEs)	142	-53	-51	-48
Capacity 200+ Mbps	Intercapital	345	-66	-64	-65
	Metro	581	-5	-1	4
	Regional	601	-35	-31	-28
	All (excl. TEs)	549	-32	-28	-26
All capacities	Intercapital	190	-59	-57	-57
	Metro	277	-11	-6	-1
	Regional	147	-54	-51	-48
	All (excl. TEs)	184	-44	-41	-38
	Tail-end	166	4	12	17
	All (incl. TEs)	182	-39	-35	-32
Deregulated					
Capacity < 5 Mbps	Intercapital	65	-10	-4	0
	Metro	10	-12	-5	0
	Regional	29	-4	5	9
	All (excl. TEs)	19	-11	-3	1
Capacity 5 - 50 Mbps	Intercapital	216	-13	-9	-8
	Metro	73	-28	-24	-21
	Regional	128	-32	-29	-23
	All (excl. TEs)	114	-24	-20	-18
Capacity 50 - 200 Mbps	Intercapital	443	-4	0	0
	Metro	272	-13	-9	-5
	Regional	196	-41	-38	-35
	All (excl. TEs)	292	-16	-12	-9
Capacity 200+ Mbps	Intercapital	1480	-26	-22	-25
	Metro	643	-4	1	4
	Regional	633	-26	-22	-19
	All (excl. TEs)	1032	-17	-13	-12
All capacities	Intercapital	1066	-20	-16	-17
	Metro	298	-13	-8	-4
	Regional	362	-29	-25	-22
	All (excl. TEs)	545	-17	-13	-11
	Tail-end	.	.	.	.
	All (incl. TEs)	545	-17	-13	-11

Capacity class	*Route type*	*DAA 2012*	*Model (2a)*	*Model (2b)*	*Models (2c)*
Regulated & Deregulated					
Capacity < 5 Mbps	Intercapital	38	-23	-17	-14
	Metro	16	-9	-1	4
	Regional	-22	-45	-40	-36
	All (excl. TEs)	-3	-29	-23	-18
Capacity 5 - 50 Mbps	Intercapital	180	-25	-22	-20
	Metro	68	-31	-27	-24
	Regional	-3	-66	-64	-61
	All (excl. TEs)	62	-43	-40	-37
Capacity 50 - 200 Mbps	Intercapital	264	-36	-33	-34
	Metro	291	-12	-8	-4
	Regional	106	-61	-59	-57
	All (excl. TEs)	193	-41	-38	-35
Capacity 200+ Mbps	Intercapital	1213	-35	-32	-34
	Metro	620	-4	0	4
	Regional	608	-33	-29	-26
	All (excl. TEs)	818	-24	-20	-18
All capacities	Intercapital	766	-33	-30	-31
	Metro	290	-12	-7	-3
	Regional	174	-51	-48	-45
	All (excl. TEs)	332	-33	-29	-26
	Tail-end	166	4	12	17
	All (incl. TEs)	322	-31	-26	-24

Source: Economic Insights estimates.

3.2 Robust Regression RE models

The term "robust regression" is used here to refer to a set of regression techniques that do not rely entirely on the least squares principle, and instead involve down-weighting (and in some cases removing) extremely influential observations using algorithms that iteratively re-weight the individual observations in the sample with successive estimations of the model. These techniques can be useful in circumstances where it is believed that most of the data in the sample is accurate, but that some of the data is contaminated, and the most extremely influential data points are considered to be those most likely to be contaminated.

In our previous analysis we tested the Stata robust regression routine *rreg*, and noted this was one of several robust regression techniques. Two others, noted by CEG, are *sregress* and *mmregress* (Verardi & Croux 2009). The techniques differ in terms of the detail of the algorithms used to re-weight the individual observations. As stated in our final report, the fact that there are various different weighting techniques (those mentioned representing just a few of them) may make this approach less attractive than other methods of reducing the influence of outliers, such as the quantile regression approach that we also tested (Economic Insights 2015, p.40). This is because the differences between the robust regression algorithms may seem to be idiosyncratic and because the choice between the techniques might be viewed as subjective.

We previously noted there is currently no available implementation in Stata (the statistical

package we used in our study) of a robust regression technique in the context of a random effects model, which is the stochastic specification of the preferred models. This is because the development and application of robust regression algorithms to random effects models is comparatively recent. CEG has observed that there is a package which implements a robust regression technique for random effects models within the R statistical software. CEG suggest that it was invalid of us not to have used this software, a criticism with which we disagree. It should be noted that R is free public domain software, which "comes with *absolutely no warranty*".[6] There is no guarantee that routines implemented in it are accurate. Unlike well-established proprietary statistical software products, which have legal liability as well as reputation to protect, freeware is unlikely to have the same degree of quality assurance and in our opinion should not be relied upon for regulatory decision-making. That said, this section provides model estimates for the robust regression RE model using the routine available in R.

The "robust estimation" of the random effects model is carried out using the 'robustlmm' user-written package for R.[7] This package implements a robust approach of fitting linear mixed effect models. It was released in August 2015 (CRAN 2015; Koller 2015) and has its origins in Manuel Koller's doctoral thesis (2013). To estimate the model using the robustlmm routine it is necessary to choose:

- the robust scoring or weight functions to be used (of which three are available)[8]
- the method used for computations (of which two are available)[9]
- tuning parameters, used within the weight functions, and chosen to trade-off robustness and efficiency,[10] and
- whether the 'robustification' of the residuals takes account of both of the stochastic elements of the model, or only of the residuals after excluding the random effects.[11]

We found it necessary to centre the data before hand in order to achieve convergence to a solution. The number of iterations needed to reach each solution was: 41 for Model (3a), 46 for Model (3b) and 30 for Model (3c). Slow convergence appears to be an intentional feature of the algorithm. The idea seems to be that as model parameters change with each iteration,

---

[6] See: < https://www r-project.org>.

[7] Two other R packages also need to be installed to run the 'robustlmm' package: 'lme4' and 'Matrix'.

[8] These are called "psi-functions" and the options available are the "classical" psi-function, the Huber psi-function, or a smoothed version of the Huber function. The smoothed Huber psi-function is used in our estimates.

[9] Both are different forms of the 'Design Adaptive Scale' methodology proposed by Koller and Stahl in a 2011 paper. These two options are 'DAStau' and 'DASvar', with the latter described as faster but less accurate. The 'DAStau' option was used in our estimates.

[10] This refers to the option to set the value of parameter 'k', which is set at 2.28 in our estimates, corresponding to efficiency of 0.95.

[11] In our estimates the 'robustification' applies to both elements.

all of the residuals change in value and their degrees of influence change, and consequently the ultimate solution can be path dependent, so "small" iteration steps are used to reduce the degree to which the ultimate solution is sensitive to the intermediate solution steps.

The results of estimating the Robust RE models are shown in Table 3.8. Because these models were estimated with centred data, it is necessary to calculate the adjustment required before the model could be used for making predictions using un-centred values of the explanatory variables. Table 3.8 also shows the adjustment required to the intercepts and the value of the intercepts after adjustment.

The adjusted intercepts are equal to:

- the estimated intercept
- plus the sample mean value of the dependent variable
- minus the sum-product of the estimated coefficients and sample means for each of the explanatory variables used in the model.

The variance components in the robust random effects model are also calculated 'robustly', which means that they are calculated based on the weighted residuals and there are also correction factors involved in their calculation. Therefore, they are not directly comparable to models in which the variance components are calculated using unweighted residuals. Because weights apply to a subset of observations and are all positive but less than or equal to one, the estimated variance components based on weighted residuals will be smaller than if they were based on unweighted residuals. Since the variance components are used when calculating the predicted monthly charges from the predicted log monthly charges — i.e. when making the adjustment relating to Jensen's inequality using the adjustment factor: $\exp((\hat{\sigma}_e^2 + \hat{\sigma}_u^2)/2)$. This implies that the predictions of monthly charges would likely be downwardly biased if the reported variance components were used for this purpose. This issue was not raised by CEG.

We have used the RMSE reported in table 3.8, using the approximation: $\exp(\hat{\sigma}_{ue}^2/2)$, where $\hat{\sigma}_{ue} = RMSE(n/(n-k))$. However, this method remains inconsistent with the adjustment factors used for the other models, where the two variance components are treated separately, and may still result in a slightly lower adjustment factor and hence lower price predictions that would be the case if the separate variance components based on unweighted residuals were known.

Table 3.8 presents the results from estimating the new random effects models with a robust regression specification. The parameters and their statistical significance are similar to the model 1 specifications in Table 2.1. The goodness-of-fit measures are also similar to model 1. The RMSE is higher and R^2 is lower for the Robust RE models, compared to the corresponding versions of model 1, while MAE for the Robust RE model is lower.

Table 3.9 shows the cost elasticities with respect to capacity and distance. The cost elasticities are all positive and their values are consistent with our broad expectations.

Table 3.10 presents the actual prices, predictions of the DAA 2012 model (as estimated by ordinary least squares) and predictions of the random effects models estimated using the

robust regression method. Table 3.11 provides a comparison of the percentage differences of the Model 3 price predictions against the DAA 2012 (or regulated prices). Table 3.12 shows the percentage differences of Model 3 predictions and DAA 2012 predictions against actual regulated and deregulated prices.

The predicted prices from all of the model 3 specifications are substantially lower than the predicted prices using the DAA 2012 model (Table 3.11).

The model 3 specifications on average predict prices for deregulated routes that are lower than the actual deregulated prices (11 to 19 per cent) while the DAA 2012 model again exhibits very poor predictive capacity particularly for higher capacity services (Table 3.12). The model 3 predictions for deregulated low capacity metro services range from 2 per cent higher than actual prices (model 3c) to 12 per cent lower than actual prices (model 3a). These results compare with the model 1 results where the overall average ranged from 9 to 18 per cent less than the actual deregulated prices and for low capacity metro services which ranged from 2 per cent higher than actual prices (model 1c) to 15 per cent less than actual prices (model 1a) (Table 2.7).

The overall average predicted price on regulated routes for the model 3 specifications range from $740 to $850 (Table 3.10). This compares with ranges for model 1 and model 2 specifications with respective ranges of $770 to $895 and $785 to $876 (Tables 2.5 and 3.4). The model 3 predictions on average range from 34 to 42 per cent less than actual prices on regulated routes (Table 3.12). This compares with the model 1 predictions of a range of 30 to 40 per cent less than average actual prices for regulated routes.

For regulated low capacity metro routes, the range for model 3 predictions is from 10 per cent above actual prices (model 3c) to 5 per cent below actual prices (model 3a), while the range for model 1 predictions is from 10 per cent above actual prices (model 2c) to 8 per cent below actual prices (model 2a).

Giving most weight to the predictions of the models for the deregulated routes the model 3 specifications involving robust regression do not seem to improve on the model 1 specifications.

Table 3.8: **Model 3: Robust RE models (2014 data)**

	Model (3a)		Model (3b)		Model (3c)	
Predictor	*Coeff.*	*t-stat.*	*Coeff.*	*t-stat.*	*Coeff.*	*t-stat.*
constant	-0.00647	-0.73	-0.00830	-0.94	-0.01060	-1.22
log capacity	0.41006	42.59	0.39816	41.37	0.40224	41.76
log distance	0.05215	3.26	0.04859	3.06	0.07266	4.32
0.5(log capacity)^2	-0.01663	-6.89	-0.01459	-6.04	-0.01562	-6.45
0.5(log distance)^2	0.02047	4.47	0.02112	4.63	0.02392	4.94
(log capacity)(log distance)	-0.00415	-3.34	-0.00402	-3.23	-0.00538	-4.41
log route t'put	-0.02353	-5.97	-0.02540	-6.47		
log ESA t'put	0.06921	10.34	0.06796	10.20		
contract start date	-0.00004	-7.72				
route class 2 (Metro)	0.02152	0.41	0.02345	0.45	0.09976	1.82
route class 3 (Regional)	0.15451	3.62	0.15393	3.61	0.16421	3.68
Provider #1	[redacted]					
Provider #3						
Provider #4						
Provider #5						
Provider #6						
Provider #7						
Provider #8						
Provider #9						
interface-type 3 (SDH)	0.27449	17.01	0.30398	19.63	0.29812	19.21
σ(u)	0.18810		0.18550		0.21260	
σ(e)	0.36320		0.36480		0.36160	
Required intercept adjustment	5.32642		4.53854		5.05340	
Intercept after adjustment	5.31995		4.53023		5.04280	
Goodness-of-fit						
R^2*	0.68015		0.67740		0.66792	
RMSE (based on ue)	0.50939		0.51155		0.51970	
MAE (based on ue)	0.35614		0.35640		0.36958	

Source: Economic Insights estimation results.

Notes: * Squared correlation between fitted and actual dependent.

Table 3.9: **Model 3: (2014 data), Cost elasticities**

	Model (3a)		Model (3b)		Model (3c)	
	capacity	*distance*	*capacity*	*distance*	*capacity*	*distance*
Deregulated routes						
10th	0.396	0.060	0.386	0.057	0.388	0.082
50th	0.352	0.092	0.346	0.091	0.344	0.118
90th	0.288	0.163	0.289	0.164	0.278	0.199
Regulated routes						
10th	0.393	0.078	0.382	0.075	0.384	0.102
50th	0.366	0.111	0.359	0.110	0.356	0.141
90th	0.302	0.153	0.301	0.154	0.291	0.188

Source: Economic Insights.

Table 3.10: **Model 3: Average predicted prices by market segment ($/month)**

Capacity class	*Route type*	*Actual*	*DAA 2012*	*Model (3a)*	*Model (3b)*	*Model (3c)*
Regulated routes						
Capacity < 5 Mbps	Intercapital	1209	1365	797	865	910
	Metro	378	458	359	394	417
	Regional	1043	782	513	563	615
	All (excl. TEs)	713	635	443	486	524
Capacity 5 - 50 Mbps	Intercapital	2881	6192	1500	1569	1626
	Metro	1074	1676	632	667	719
	Regional	3995	3409	1089	1162	1313
	All (excl. TEs)	2385	2807	897	950	1041
Capacity 50 - 200 Mbps	Intercapital	6677	13969	2333	2452	2431
	Metro	1524	6405	1272	1329	1433
	Regional	6129	11946	1937	2033	2247
	All (excl. TEs)	3818	9253	1624	1701	1844
Capacity 200+ Mbps	Intercapital	11784	52403	4029	4226	4174
	Metro	2023	13770	1835	1918	2054
	Regional	5198	36426	3060	3228	3494
	All (excl. TEs)	3834	24883	2442	2565	2741
All capacities	Intercapital	4437	12861	1793	1892	1910
	Metro	824	3106	707	752	804
	Regional	2472	6116	1024	1094	1200
	All (excl. TEs)	1718	4877	896	954	1029
	Tail-end	[redacted]	1101	426	462	487
	All (incl. TEs)	[redacted]	3627	740	791	850
Deregulated						
Capacity < 5 Mbps	Intercapital	812	1340	747	808	842
	Metro	398	440	351	385	405
	Regional	504	653	460	505	535
	All (excl. TEs)	434	516	386	423	445
Capacity 5 - 50 Mbps	Intercapital	1534	4853	1299	1362	1403
	Metro	859	1489	602	637	670
	Regional	1884	4294	1106	1165	1318
	All (excl. TEs)	1010	2161	738	779	817
Capacity 50 - 200 Mbps	Intercapital	2432	13215	2245	2348	2385
	Metro	1399	5202	1161	1215	1288
	Regional	3475	10279	1783	1870	2051
	All (excl. TEs)	1714	6723	1361	1425	1502
Capacity 200+ Mbps	Intercapital	7621	120388	5840	6191	6014
	Metro	1979	14698	1874	1962	2062
	Regional	4449	32605	2968	3133	3359
	All (excl. TEs)	3405	38549	2811	2962	3017
All capacities	Intercapital	3290	38367	2668	2821	2798
	Metro	923	3673	786	831	876
	Regional	2009	9274	1283	1361	1471
	All (excl. TEs)	1307	8433	1060	1122	1164
	Tail-end	.	.	.	.	.
	All (incl. TEs)	1307	8433	1060	1122	1164

Capacity class	*Route type*	*Actual*	*DAA 2012*	*Model (3a)*	*Model (3b)*	*Model (3c)*
Regulated & Deregulated						
Capacity < 5 Mbps	Intercapital	979	1350	768	832	871
	Metro	388	449	355	389	411
	Regional	980	767	507	556	606
	All (excl. TEs)	607	590	422	462	494
Capacity 5 - 50 Mbps	Intercapital	1842	5160	1345	1409	1454
	Metro	911	1535	609	644	681
	Regional	3667	3546	1091	1163	1314
	All (excl. TEs)	1465	2374	791	835	891
Capacity 50 - 200 Mbps	Intercapital	3689	13438	2271	2379	2399
	Metro	1445	5642	1202	1257	1341
	Regional	5645	11642	1909	2003	2211
	All (excl. TEs)	2700	7909	1484	1554	1663
Capacity 200+ Mbps	Intercapital	8311	109128	5540	5866	5710
	Metro	1995	14363	1860	1946	2059
	Regional	4998	35407	3035	3203	3458
	All (excl. TEs)	3583	32878	2658	2797	2902
All capacities	Intercapital	3610	31261	2424	2562	2551
	Metro	882	3439	753	798	847
	Regional	2404	6581	1062	1133	1240
	All (excl. TEs)	1521	6579	975	1035	1094
	Tail-end	[redacted]	1101	426	462	487
	All (incl. TEs)	[redacted]	5455	862	917	969

Note: the DAA 2012 model is not re-estimated with robust regression.

Table 3.11: **Model 3: Difference in predicted prices compared to DAA 2012 (%)**

Capacity class	*Route type*	*Model (3a)*	*Model (3b)*	*Model (3c)*
Regulated routes				
Capacity < 5 Mbps	Intercapital	-42	-37	-33
	Metro	-22	-14	-9
	Regional	-34	-28	-21
	All (excl. TEs)	-30	-23	-18
Capacity 5 - 50 Mbps	Intercapital	-76	-75	-74
	Metro	-62	-60	-57
	Regional	-68	-66	-61
	All (excl. TEs)	-68	-66	-63
Capacity 50 - 200 Mbps	Intercapital	-83	-82	-83
	Metro	-80	-79	-78
	Regional	-84	-83	-81
	All (excl. TEs)	-82	-82	-80
Capacity 200+ Mbps	Intercapital	-92	-92	-92
	Metro	-87	-86	-85
	Regional	-92	-91	-90
	All (excl. TEs)	-90	-90	-89
All capacities	Intercapital	-86	-85	-85
	Metro	-77	-76	-74
	Regional	-83	-82	-80
	All (excl. TEs)	-82	-80	-79

Capacity class	*Route type*	*Model (3a)*	*Model (3b)*	*Model (3c)*
	Tail-end	-61	-58	-56
	All (incl. TEs)	-80	-78	-77
Deregulated				
Capacity < 5 Mbps	Intercapital	-44	-40	-37
	Metro	-20	-12	-8
	Regional	-30	-23	-18
	All (excl. TEs)	-25	-18	-14
Capacity 5 - 50 Mbps	Intercapital	-73	-72	-71
	Metro	-60	-57	-55
	Regional	-74	-73	-69
	All (excl. TEs)	-66	-64	-62
Capacity 50 - 200 Mbps	Intercapital	-83	-82	-82
	Metro	-78	-77	-75
	Regional	-83	-82	-80
	All (excl. TEs)	-80	-79	-78
Capacity 200+ Mbps	Intercapital	-95	-95	-95
	Metro	-87	-87	-86
	Regional	-91	-90	-90
	All (excl. TEs)	-93	-92	-92
All capacities	Intercapital	-93	-93	-93
	Metro	-79	-77	-76
	Regional	-86	-85	-84
	All (excl. TEs)	-87	-87	-86
	Tail-end	.	.	.
	All (incl. TEs)	-87	-87	-86
Regulated & Deregulated				
Capacity < 5 Mbps	Intercapital	-43	-38	-36
	Metro	-21	-13	-8
	Regional	-34	-27	-21
	All (excl. TEs)	-28	-22	-16
Capacity 5 - 50 Mbps	Intercapital	-74	-73	-72
	Metro	-60	-58	-56
	Regional	-69	-67	-63
	All (excl. TEs)	-67	-65	-62
Capacity 50 - 200 Mbps	Intercapital	-83	-82	-82
	Metro	-79	-78	-76
	Regional	-84	-83	-81
	All (excl. TEs)	-81	-80	-79
Capacity 200+ Mbps	Intercapital	-95	-95	-95
	Metro	-87	-86	-86
	Regional	-91	-91	-90
	All (excl. TEs)	-92	-91	-91
All capacities	Intercapital	-92	-92	-92
	Metro	-78	-77	-75
	Regional	-84	-83	-81
	All (excl. TEs)	-85	-84	-83
	Tail-end	-61	-58	-56
	All (incl. TEs)	-84	-83	-82

Table 3.12: **Model 3: Difference in predicted prices compared to actual (%)**

Capacity class	*Route type*	*DAA 2012*	*Model (3a)*	*Model (3b)*	*Model (3c)*
Regulated routes					
Capacity < 5 Mbps	Intercapital	13	-34	-28	-25
	Metro	21	-5	4	10
	Regional	-25	-51	-46	-41
	All (excl. TEs)	-11	-38	-32	-27
Capacity 5 - 50 Mbps	Intercapital	115	-48	-46	-44
	Metro	56	-41	-38	-33
	Regional	-15	-73	-71	-67
	All (excl. TEs)	18	-62	-60	-56
Capacity 50 - 200 Mbps	Intercapital	109	-65	-63	-64
	Metro	320	-17	-13	-6
	Regional	95	-68	-67	-63
	All (excl. TEs)	142	-57	-55	-52
Capacity 200+ Mbps	Intercapital	345	-66	-64	-65
	Metro	581	-9	-5	2
	Regional	601	-41	-38	-33
	All (excl. TEs)	549	-36	-33	-29
All capacities	Intercapital	190	-60	-57	-57
	Metro	277	-14	-9	-2
	Regional	147	-59	-56	-51
	All (excl. TEs)	184	-48	-44	-40
	Tail-end	166	3	12	18
	All (incl. TEs)	182	-42	-38	-34
Deregulated					
Capacity < 5 Mbps	Intercapital	65	-8	-1	4
	Metro	10	-12	-3	2
	Regional	29	-9	0	6
	All (excl. TEs)	19	-11	-3	2
Capacity 5 - 50 Mbps	Intercapital	216	-15	-11	-8
	Metro	73	-30	-26	-22
	Regional	128	-41	-38	-30
	All (excl. TEs)	114	-27	-23	-19
Capacity 50 - 200 Mbps	Intercapital	443	-8	-3	-2
	Metro	272	-17	-13	-8
	Regional	196	-49	-46	-41
	All (excl. TEs)	292	-21	-17	-12
Capacity 200+ Mbps	Intercapital	1480	-23	-19	-21
	Metro	643	-5	-1	4
	Regional	633	-33	-30	-25
	All (excl. TEs)	1032	-17	-13	-11
All capacities	Intercapital	1066	-19	-14	-15
	Metro	298	-15	-10	-5
	Regional	362	-36	-32	-27
	All (excl. TEs)	545	-19	-14	-11
	Tail-end	.	.	.	.
	All (incl. TEs)	545	-19	-14	-11

Capacity class	*Route type*	*DAA 2012*	*Model (3a)*	*Model (3b)*	*Model (3c)*
Regulated & Deregulated					
Capacity < 5 Mbps	Intercapital	38	-22	-15	-11
	Metro	16	-9	0	6
	Regional	-22	-48	-43	-38
	All (excl. TEs)	-3	-31	-24	-19
Capacity 5 - 50 Mbps	Intercapital	180	-27	-23	-21
	Metro	68	-33	-29	-25
	Regional	-3	-70	-68	-64
	All (excl. TEs)	62	-46	-43	-39
Capacity 50 - 200 Mbps	Intercapital	264	-38	-36	-35
	Metro	291	-17	-13	-7
	Regional	106	-66	-65	-61
	All (excl. TEs)	193	-45	-42	-38
Capacity 200+ Mbps	Intercapital	1213	-33	-29	-31
	Metro	620	-7	-2	3
	Regional	608	-39	-36	-31
	All (excl. TEs)	818	-26	-22	-19
All capacities	Intercapital	766	-33	-29	-29
	Metro	290	-15	-10	-4
	Regional	174	-56	-53	-48
	All (excl. TEs)	332	-36	-32	-28
	Tail-end	166	3	12	18
	All (incl. TEs)	322	-33	-29	-25

Source: Economic Insights estimates.

4 THE INFLUENCE OF 2 MBPS SERVICES

Optus and CEG have raised concerns about the predictive ability of the preferred econometric pricing models developed in our final report, for services of 2 Mbps, or thereabouts. Previously we tested whether an indicator variable for 2 Mbps services should be included in the model under the hypothesis that there may be some additional costs associated with these services, perhaps associated with "bundled tail-ends". However, we found insufficient support for including a variable so defined.[12] CEG suggested an alternative indicator variable for 2 Mbps services of less than 5 km distance could be used. They state that including a variable so defined would "go some way to addressing our concerns, however the gap between predicted and actual (commercially negotiated) prices remains significant" (CEG 2015, p.3). Optus suggested the prices for 2 Mbps services over short distances should be determined by a different method, and not using the econometric model, and they should be no higher than actual prices on regulated routes.

In this section we test several different methods of addressing the possibility of systematic differences in charges for Mbps services compared to other DTCS services. The methods tested are:

- excluding all services that are less than 2.5 Mbps and also less than 5 km in distance from the sample of data used for estimation (section 4.1)
- retaining 2 Mbps services in the sample, and including a specific effect relating to these services (section 4.2).
- developing a piecewise regression model with a knot at 2.5 Mbps (section 4.3).

4.1 Excluding 2.5 Mbps Services

To test Optus' proposal that a different method be used to determine 2 Mbps services, we estimated models with the same specifications as Model 1, but using two separate subsamples:

(i) All deregulated services excluding services of less than 2.5 Mbps and less than 5 km, and

(ii) Only those services of less than 2.5 Mbps and less than 5 km.

Econometric Results

The models estimated using data sample (i) — i.e. based on a reduced sample which excludes services of less than 2.5 Mbps and less than 5 km — are shown as Models (4a) to (4c) in Table 4.1. The model specifications are the same as Model 1. Table 4.2 shows the test statistics for the Models shown in Table 4.1.

[12] The previous analysis used a dummy variable for services with capacity equal to 2 Mbps. However, because there are also many services with capacity slightly greater than 2 Mbps, the analysis here includes all services of less than 2.5 Mbps as "2 Mbps services".

To attempt to make the goodness-of-fit statistics comparable with those of Model 1, they are calculated by combining the observations of sample types (i) and (ii) and using the applicable predictions (and associated prediction errors) being:

- the econometric models shown in Table 4.1 for all deregulated services excluding services of less than 2.5 Mbps and less than 5 km, and
- the mean value of log monthly charges for services of less than 2.5 Mbps and less than 5 km on deregulated routes for all services of that type.

Table 4.1: **Model 4: RE models excl. 2 Mbps services* (2014 data)**

	Model (4a)		**Model (4b)**		**Model (4c)**	
Predictor	*Coeff.*	*t-stat.*	*Coeff.*	*t-stat.*	*Coeff.*	*t-stat.*
constant	7.22395	37.00	5.18957	38.88	5.37745	59.21
log capacity	0.45543	36.38	0.44146	34.85	0.43661	34.59
log distance	-0.06938	-2.80	-0.07624	-3.03	-0.04788	-1.91
0.5(log capacity)^2	-0.03143	-10.63	-0.02959	-9.87	-0.02875	-9.60
0.5(log distance)^2	0.04389	6.79	0.04508	6.86	0.04422	6.67
(log capacity)(log distance)	-0.00018	-0.11	-0.00015	-0.09	-0.00192	-1.19
log route t'put	-0.02916	-5.29	-0.03233	-5.77	.	.
log ESA t'put	0.04903	4.74	0.03715	3.54	.	.
contract start date	-0.00011	-14.08	.	.	.	.
route class 2 (Metro)	0.03776	0.53	0.03249	0.45	0.08145	1.12
route class 3 (Regional)	0.22374	4.00	0.21135	3.72	0.22710	3.95
Provider #1	[redacted]					
Provider #3						
Provider #4						
Provider #5						
Provider #6						
Provider #7						
Provider #8						
Provider #9						
interface-type 3 (SDH)	0.20734	9.92	0.30084	14.95	0.29929	14.87
σ(u)	0.29436		0.29957		0.30821	
σ(e)	0.43680		0.44337		0.44288	

Source: Economic Insights estimation results.

Notes: * "2 Mbps services" here refers to services of less than 2.5 Mbps and less than 5 km.

Initially, econometric models were tested using the second of the data samples, namely for short metro "2 Mbps" services. The second-order output variables were initially found to be insignificant, and when removed the main output effects were also insignificant. Indeed the estimates suggested that none of the explanatory variables were significant except some provider fixed-effects. Given the lack of explanatory variables, the mean value of the log monthly charge for services of less than 2.5 Mbps and less than 5 km on deregulated routes is used as the best estimate for forecasting log monthly charges of services in that category, and the standard deviation is used in place of the standard error of prediction, and when deriving the predicted price for these services.

The predicted prices for these short, low capacity, metro services, is calculated using the antilog of the average log monthly charge just mentioned, and with an adjustment for Jensen's inequality using the standard deviation.

Table 4.2: **Model 4: RE models excl. 2 Mbps services* (2014 data): Statistical tests**

	Model (4a)		Model (4b)		Model (4c)	
	Stat.	*P-value***	*Stat.*	*P-value***	*Stat.*	*P-value***
Normality of residuals						
Doornik-Hansen(1)	3663.9	0.0000	3711.2	0.0000	3629.0	0.0000
IQR (% severe outliers) (2)†	1.83%		2.22%		2.35%	
Homoscedasticity						
Breusch-Pagan/Cook-Weisberg(6)	802.0[a]	0.0000	812.3[a]	0.0000	926.1[a]	0.0000
Multicollinearity						
# VIF scores > 10	5/19		5/18		5/16	
Misspecification						
RESET (7)	28.0[a]	0.0000	7.6[a]	0.0000	7.8[a]	0.0000
Link test(8)	3.76[a]	0.000	1.08[a]	0.282	0.76[a]	0.446
Joint significance tests						
2nd order output terms (df = 3)	169.8	0.0000	153.9	0.0000	152.0	0.0000
Route classes (df = 2)	36.8	0.0000	32.4	0.0000	28.2	0.0000
Provider fixed effects (df = 7)	861.2	0.0000	853.6	0.0000	852.8	0.0000

Note: * "2 Mbps services" here refers to services of less than 2.5 Mbps and less than 5 km. ** Null hypothesis is rejected, as a standard procedure, in these tests, if P-value is less than 0.05. Equivalently, the reported statistic exceeds the critical value for that statistic; † Percentage of n = 7,708 observations; (1) $chi^2(2k)$ where k = 19 for 1st model, and k = 18 for 2nd model and k = 16 for 3rd model. (2) Severe outliers represent about 0.0002% of a normal distribution; (3) Studentized residual > 3; (4) Hat value > $3k/n$; (5) Cook's D > 5 × average Cook's D; (6) $chi^2(1)$; (7) Via powers of the dependent variable, $F(3,n–k–3)$; (8) t-statistic on hat^2; (9) $F(r,n–k–r)$, where r = number of parameters tested, and r = 3 for higher-order output terms, r = 2 for route classes, and r = 8 for provider-specific effects. (10) $chibar^2(1)$; a Approximate, based on OLS regression of $(y - \hat{u}_i)$ on the predictors.

The goodness-of-fit statistics in Table 4.3 show the combined effect of these two prediction methods for subsamples (i) and (ii) above, and can be directly compared against those for Model 1 because the combined samples are the same (whether within-sample, validation sample or full sample). The BIC statistics are not shown because they are not available on a comparable basis to those reported for Model 1.

Table 4.3 shows that the goodness-of-fit measures for Model 4, using a different prediction method for observations less than 2.5 Mbps and less than 5 km, are comparable to those for Model 1.

Table 4.3: **Model 4: Goodness-of-fit**

	Model (4a)		**Model (4b)**		**Model (4c)**	
Goodness-of-fit statistic	*10-fold validation (avg)*	*Full sample*	*10-fold validation (avg)*	*Full sample*	*10-fold validation (avg)*	*Full sample*
Within-sample						
obs	6,937[a]	7,708[a]	6,937[a]	7,708[a]	6,937[a]	7,708[a]
R^2*	0.69620	0.69585	0.68913	0.68877	0.68438	0.68410
BIC						
RMSE (based on ue)	0.49640	0.49674	0.50203	0.50237	0.50705	0.50746
MAE (based on ue)	0.35772	0.35796	0.36045	0.36072	0.36522	0.36550
Out-of-sample						
obs	771[a]	0	771[a]	0	771[a]	0
R^2*	0.69411	.	0.68703	.	0.68253	.
RMSE (based on ue)	0.49818	.	0.50373	.	0.50852	.
MAE (based on ue)	0.35894	.	0.36156	.	0.36624	.

Notes: * "2 Mbps services" here refers to services of less than 2.5 Mbps and less than 5 km. ** Squared correlation between fitted and actual dependent. [a] Goodness-of-fit statistics are calculated using model predictions for all services excluding "2 Mbps services" and using the mean of log monthly charge for "2 Mbps services" on deregulated routes as the predicted prices for all "2 Mbps services".

Elasticities of Cost with Respect to Each Output

Table 4.4 presents the elasticities of cost with respect to capacity and distance for each of the three models presented in Table 4.1. The elasticity of cost to capacity diminishes at higher levels of capacity. The elasticity of cost to distance increases with higher distances.

Table 4.4: **Model 4: Base case RE models (2014 data): Cost elasticities**

	Model (4a)		**Model (4b)**		**Model (4c)**	
	capacity	*distance*	*capacity*	*distance*	*capacity*	*distance*
Deregulated routes						
10th	0.434	-0.046	0.421	-0.052	0.416	-0.025
50th	0.365	0.041	0.357	0.038	0.350	0.059
90th	0.275	0.218	0.272	0.219	0.260	0.231
Regulated routes						
10th	0.433	-0.008	0.421	-0.013	0.414	0.013
50th	0.398	0.073	0.387	0.071	0.378	0.093
90th	0.296	0.191	0.291	0.191	0.280	0.205

Source: Economic Insights.

Average Price Effects per Market Segment

Table 4.5 shows the actual average prices and the average predicted prices per market segment associated with the DAA 2012 Model and the Model 4 specifications (including the predicted values of small capacity metro services using the separate method, as explained). These averages are calculated over all observations, including those with less than 2.5 Mbps and 5 km in distance.

Table 4.6 shows the comparison of the average predicted prices from the models against the prices predicted by the DAA 2012 model in each market segment. It can be noted that the average predicted prices of Models (4b) and (4c) for regulated metro services of between 2.5 and 5 Mbps are 9 per cent and 6 per cent below the current regulated price (as estimated with the DAA 2012 model), respectively.

Table 4.7 shows the comparison between the average predicted prices from the models and actual prices for both deregulated and regulated routes. The model 4 specifications on average predict prices for deregulated routes that range from 0 per cent difference relative to actual prices (models 4b and 4c) to 13 per cent less than actual prices for deregulated routes. For metro routes of less than 5 Mbps capacity the range is from 5 per cent higher than the actual deregulated price (model 4c) to 12 per cent less than the actual deregulated price (model 4a).

These results compare with the model 1 results where the overall average ranged from 9 to 18 per cent less than the actual deregulated prices and for low capacity metro services which ranged from 2 per cent higher than actual prices (model 1c) to 15 per cent less than actual prices (model 1a) (Table 2.7).

Turning to the predictions for regulated routes, the overall average predicted price on regulated routes for the model 4 specifications range from $810 to $990 (Table 4.5). This compares with ranges for model 1 and model 2 specifications with respective ranges of $770 to $895 and $785 to $876 (Tables 2.5 and 3.4).

For regulated low capacity metro routes, the range for model 4 predictions is from 14 per cent above actual prices (model 4c) to 7 per cent below actual prices (model 4a), while the range for model 1 predictions is from 9 per cent above actual prices (model 2c) to 5 per cent below actual prices (model 2a) (Table 3.6).

However, the model 4 predictions for tail end services are substantially higher for model 1. The model 4 predictions for tail end services range from $518 to $660 whereas the range for model 1 was $429 to $505 (Table 2.5).

Giving most weight to the predictions of the models for the deregulated routes the model 4 specifications appear to offer a small improvement on the model 1 specifications but the predictions for tail end services may be too high.

Table 4.5: **Model 4: Average predicted prices by market segment ($/month)**

Capacity class	*Route type*	*Actual*	*DAA 2012*	*Model (4a)*	*Model (4b)*	*Models (4c)*
Regulated routes						
Capacity < 5 Mbps	Intercapital	1209	1365	738	914	959
	Metro	378	458	353	415	431
	Regional	1043	782	509	639	676
	All (excl. TEs)	713	635	437	534	561
Capacity 5 - 50 Mbps	Intercapital	2881	6192	1615	1827	1847
	Metro	1074	1676	696	806	841
	Regional	3995	3409	1221	1473	1558
	All (excl. TEs)	2385	2807	993	1168	1220
Capacity 50 - 200 Mbps	Intercapital	6677	13969	2413	2788	2728
	Metro	1524	6405	1396	1584	1638
	Regional	6129	11946	2237	2587	2666
	All (excl. TEs)	3818	9253	1820	2089	2146
Capacity 200+ Mbps	Intercapital	11784	52403	4200	4759	4583
	Metro	2023	13770	1952	2206	2268
	Regional	5198	36426	3363	3902	3948
	All (excl. TEs)	3834	24883	2635	3018	3060
All capacities	Intercapital	4437	12861	1839	2127	2109
	Metro	824	3106	744	853	883
	Regional	2472	6116	1108	1320	1369
	All (excl. TEs)	1718	4877	954	1118	1154
	Tail-end	[redacted]	1101	518	625	660
	All (incl. TEs)	[redacted]	3627	810	955	990
Deregulated						
Capacity < 5 Mbps	Intercapital	812	1340	688	842	879
	Metro	398	440	351	403	417
	Regional	504	653	453	554	576
	All (excl. TEs)	434	516	382	445	461
Capacity 5 - 50 Mbps	Intercapital	1534	4853	1362	1549	1569
	Metro	859	1489	715	831	846
	Regional	1884	4294	1293	1501	1580
	All (excl. TEs)	1010	2161	845	977	995
Capacity 50 - 200 Mbps	Intercapital	2432	13215	2415	2743	2715
	Metro	1399	5202	1331	1515	1546
	Regional	3475	10279	2048	2366	2431
	All (excl. TEs)	1714	6723	1539	1754	1780
Capacity 200+ Mbps	Intercapital	7621	120388	5703	6541	6217
	Metro	1979	14698	2038	2310	2343
	Regional	4449	32605	3213	3750	3784
	All (excl. TEs)	3405	38549	2921	3338	3297
All capacities	Intercapital	3290	38367	2674	3068	2985
	Metro	923	3673	867	990	1011
	Regional	2009	9274	1397	1638	1676
	All (excl. TEs)	1307	8433	1135	1302	1311
	Tail-end	.	.	.	.	.
	All (incl. TEs)	1307	8433	1135	1302	1311

Capacity class	*Route type*	*Actual*	*DAA 2012*	*Model (4a)*	*Model (4b)*	*Models (4c)*
Regulated & Deregulated						
Capacity < 5 Mbps	Intercapital	979	1350	709	872	913
	Metro	388	449	352	409	424
	Regional	980	767	503	629	665
	All (excl. TEs)	607	590	416	500	523
Capacity 5 - 50 Mbps	Intercapital	1842	5160	1420	1613	1632
	Metro	911	1535	710	825	845
	Regional	3667	3546	1232	1478	1561
	All (excl. TEs)	1465	2374	894	1040	1070
Capacity 50 - 200 Mbps	Intercapital	3689	13438	2414	2756	2719
	Metro	1445	5642	1355	1540	1580
	Regional	5645	11642	2202	2547	2623
	All (excl. TEs)	2700	7909	1671	1911	1952
Capacity 200+ Mbps	Intercapital	8311	109128	5454	6246	5946
	Metro	1995	14363	2007	2272	2316
	Regional	4998	35407	3323	3862	3905
	All (excl. TEs)	3583	32878	2802	3205	3198
All capacities	Intercapital	3610	31261	2442	2806	2741
	Metro	882	3439	816	934	958
	Regional	2404	6581	1150	1367	1414
	All (excl. TEs)	1521	6579	1041	1206	1229
	Tail-end	[redacted]	1101	518	625	660
	All (incl. TEs)	[redacted]	5455	933	1087	1112

Table 4.6: **Model 4: Difference in predicted prices compared to DAA 2012 (%)**

Capacity class	*Route type*	*Model (4a)*	*Model (4b)*	*Models (4c)*
Regulated routes				
Capacity < 5 Mbps	Intercapital	-46	-33	-30
	Metro	-23	-9	-6
	Regional	-35	-18	-13
	All (excl. TEs)	-31	-16	-12
Capacity 5 - 50 Mbps	Intercapital	-74	-70	-70
	Metro	-58	-52	-50
	Regional	-64	-57	-54
	All (excl. TEs)	-65	-58	-57
Capacity 50 - 200 Mbps	Intercapital	-83	-80	-80
	Metro	-78	-75	-74
	Regional	-81	-78	-78
	All (excl. TEs)	-80	-77	-77
Capacity 200+ Mbps	Intercapital	-92	-91	-91
	Metro	-86	-84	-84
	Regional	-91	-89	-89
	All (excl. TEs)	-89	-88	-88
All capacities	Intercapital	-86	-83	-84
	Metro	-76	-73	-72
	Regional	-82	-78	-78
	All (excl. TEs)	-80	-77	-76

Capacity class	*Route type*	*Model (4a)*	*Model (4b)*	*Models (4c)*
	Tail-end	-53	-43	-40
	All (incl. TEs)	-78	-74	-73
Deregulated				
Capacity < 5 Mbps	Intercapital	-49	-37	-34
	Metro	-20	-8	-5
	Regional	-31	-15	-12
	All (excl. TEs)	-26	-14	-11
Capacity 5 - 50 Mbps	Intercapital	-72	-68	-68
	Metro	-52	-44	-43
	Regional	-70	-65	-63
	All (excl. TEs)	-61	-55	-54
Capacity 50 - 200 Mbps	Intercapital	-82	-79	-79
	Metro	-74	-71	-70
	Regional	-80	-77	-76
	All (excl. TEs)	-77	-74	-74
Capacity 200+ Mbps	Intercapital	-95	-95	-95
	Metro	-86	-84	-84
	Regional	-90	-88	-88
	All (excl. TEs)	-92	-91	-91
All capacities	Intercapital	-93	-92	-92
	Metro	-76	-73	-72
	Regional	-85	-82	-82
	All (excl. TEs)	-87	-85	-84
	Tail-end	.	.	.
	All (incl. TEs)	-87	-85	-84
Regulated & Deregulated				
Capacity < 5 Mbps	Intercapital	-47	-35	-32
	Metro	-22	-9	-5
	Regional	-34	-18	-13
	All (excl. TEs)	-29	-15	-11
Capacity 5 - 50 Mbps	Intercapital	-72	-69	-68
	Metro	-54	-46	-45
	Regional	-65	-58	-56
	All (excl. TEs)	-62	-56	-55
Capacity 50 - 200 Mbps	Intercapital	-82	-79	-80
	Metro	-76	-73	-72
	Regional	-81	-78	-77
	All (excl. TEs)	-79	-76	-75
Capacity 200+ Mbps	Intercapital	-95	-94	-95
	Metro	-86	-84	-84
	Regional	-91	-89	-89
	All (excl. TEs)	-91	-90	-90
All capacities	Intercapital	-92	-91	-91
	Metro	-76	-73	-72
	Regional	-83	-79	-79
	All (excl. TEs)	-84	-82	-81
	Tail-end	-53	-43	-40
	All (incl. TEs)	-83	-80	-80

Table 4.7: **Model 4: Difference in predicted prices compared to Actual (%)**

Capacity class	*Route type*	*DAA 2012*	*Model (4a)*	*Model (4b)*	*Models (4c)*
Regulated routes					
Capacity < 5 Mbps	Intercapital	13	-39	-24	-21
	Metro	21	-7	10	14
	Regional	-25	-51	-39	-35
	All (excl. TEs)	-11	-39	-25	-21
Capacity 5 - 50 Mbps	Intercapital	115	-44	-37	-36
	Metro	56	-35	-25	-22
	Regional	-15	-69	-63	-61
	All (excl. TEs)	18	-58	-51	-49
Capacity 50 - 200 Mbps	Intercapital	109	-64	-58	-59
	Metro	320	-8	4	7
	Regional	95	-64	-58	-56
	All (excl. TEs)	142	-52	-45	-44
Capacity 200+ Mbps	Intercapital	345	-64	-60	-61
	Metro	581	-4	9	12
	Regional	601	-35	-25	-24
	All (excl. TEs)	549	-31	-21	-20
All capacities	Intercapital	190	-59	-52	-52
	Metro	277	-10	4	7
	Regional	147	-55	-47	-45
	All (excl. TEs)	184	-44	-35	-33
	Tail-end	166	25	51	59
	All (incl. TEs)	182	-37	-26	-23
Deregulated					
Capacity < 5 Mbps	Intercapital	65	-15	4	8
	Metro	10	-12	1	5
	Regional	29	-10	10	14
	All (excl. TEs)	19	-12	3	6
Capacity 5 - 50 Mbps	Intercapital	216	-11	1	2
	Metro	73	-17	-3	-2
	Regional	128	-31	-20	-16
	All (excl. TEs)	114	-16	-3	-1
Capacity 50 - 200 Mbps	Intercapital	443	-1	13	12
	Metro	272	-5	8	10
	Regional	196	-41	-32	-30
	All (excl. TEs)	292	-10	2	4
Capacity 200+ Mbps	Intercapital	1480	-25	-14	-18
	Metro	643	3	17	18
	Regional	633	-28	-16	-15
	All (excl. TEs)	1032	-14	-2	-3
All capacities	Intercapital	1066	-19	-7	-9
	Metro	298	-6	7	9
	Regional	362	-30	-18	-17
	All (excl. TEs)	545	-13	0	0
	Tail-end	.	.	.	.
	All (incl. TEs)	545	-13	0	0

Capacity class	*Route type*	*DAA 2012*	*Model (4a)*	*Model (4b)*	*Models (4c)*
Regulated & Deregulated					
Capacity < 5 Mbps	Intercapital	38	-28	-11	-7
	Metro	16	-9	5	9
	Regional	-22	-49	-36	-32
	All (excl. TEs)	-3	-32	-18	-14
Capacity 5 - 50 Mbps	Intercapital	180	-23	-12	-11
	Metro	68	-22	-9	-7
	Regional	-3	-66	-60	-57
	All (excl. TEs)	62	-39	-29	-27
Capacity 50 - 200 Mbps	Intercapital	264	-35	-25	-26
	Metro	291	-6	7	9
	Regional	106	-61	-55	-54
	All (excl. TEs)	193	-38	-29	-28
Capacity 200+ Mbps	Intercapital	1213	-34	-25	-28
	Metro	620	1	14	16
	Regional	608	-34	-23	-22
	All (excl. TEs)	818	-22	-11	-11
All capacities	Intercapital	766	-32	-22	-24
	Metro	290	-8	6	9
	Regional	174	-52	-43	-41
	All (excl. TEs)	332	-32	-21	-19
	Tail-end	166	25	51	59
	All (incl. TEs)	322	-28	-16	-14

4.2 Including a Specific Effect for 2.5 Mbps Services

This section presents the results of introducing an indicator (or dummy) variable for short distance services (less than 5 km) of less than 2.5 Mbps.

Econometric Results

The econometric estimates are shown in Table 4.7. The coefficient on the 2.5 Mbps indicator is negative and large in absolute value, and highly statistically significant. The most notable change to the other coefficients is that the coefficient on log distance is negative and not significantly different from zero. The coefficient on the squared value of log distance remains positive and significant.

The goodness-of-fit measures shown in Table 4.8 generally indicate a slight improvement over Model 1. This includes a small improvement in the BIC relative to models (1b) and (1c) (but not for model (1a)). The statistical test results shown in Table 4.9 are comparable to those for Model 1, which were discussed in section 2.1.

Table 4.8: **Model 5: RE models with 2.5 Mbps dummy (2015 data)**

	Model (5a)		**Model (5b)**		**Model (5c)**	
Predictor	*Coeff.*	*t-stat.*	*Coeff.*	*t-stat.*	*Coeff.*	*t-stat.*
constant	6.00204	37.34	4.86336	43.92	5.20213	62.60
log capacity	0.42972	36.61	0.42237	35.84	0.41945	35.64
log distance	-0.02144	-0.95	-0.02513	-1.10	-0.00305	-0.13
0.5(log capacity)^2	-0.02748	-9.84	-0.02650	-9.44	-0.02596	-9.25
0.5(log distance)^2	0.03429	5.64	0.03490	5.69	0.03594	5.77
(log capacity)(log distance)	0.00042	0.27	0.00044	0.28	-0.00095	-0.63
log route t'put	-0.02509	-4.98	-0.02761	-5.44	.	.
log ESA t'put	0.05292	6.11	0.04943	5.66	.	.
contract start date	-0.00006	-9.69	.	.	.	.
route class 2 (Metro)	0.07173	1.06	0.07033	1.03	0.12711	1.85
route class 3 (Regional)	0.24360	4.51	0.23675	4.35	0.24931	4.52
Provider #1						
Provider #3						
Provider #4						
Provider #5						
Provider #6						
Provider #7						
Provider #8						
Provider #9						
interface-type 3 (SDH)	0.26109	13.97	0.31214	17.31	0.30970	17.15
2 Mbps dummy	-0.23425	-9.66	-0.22727	-9.31	-0.23503	-9.60
σ(u)	0.28945		0.29300		0.30232	
σ(e)	0.41672		0.41892		0.41838	

Source: Economic Insights estimation results.

Table 4.9: **Model 5: RE models with 2.5 Mbps dummy (2015 data): Goodness-of-fit**

	Model (5a)		**Model (5b)**		**Model (5c)**	
Goodness-of-fit statistic	*10-fold validation (avg)*	*Full sample*	*10-fold validation (avg)*	*Full sample*	*10-fold validation (avg)*	*Full sample*
obs	6,937	7,708	6,937	7,708	6,937	7,708
Within-sample						
R^2*	0.69010	0.68965	0.68702	0.68654	0.68057	0.68015
BIC		10,085.9		10,170.4		10,192.5
RMSE (based on ue)	0.50151	0.50197	0.50392	0.50439	0.50975	0.51024
MAE (based on ue)	0.35574	0.35620	0.35711	0.35764	0.36536	0.36571
Out-of-sample						
R^2*	0.68813	.	0.68507	.	0.67879	.
RMSE (based on ue)	0.50311	.	0.50545	.	0.51110	.
MAE (based on ue)	0.35689	.	0.35823	.	0.36634	.

Notes: * Squared correlation between fitted and actual dependent.

Table 4.10: **Model 5: RE models with 2.5 Mbps dummy (2015 data): Statistical tests**

	Model (5a)		Model (5b)		Model (5c)	
	Stat.	*P-value**	*Stat.*	*P-value**	*Stat.*	*P-value**
Normality of residuals						
Doornik-Hansen(1)	4845.9	0.0000	4971.6	0.0000	4882.2	0.0000
IQR (% severe outliers) (2)†	2.24%		2.63%		2.73%	
Homoscedasticity						
Breusch-Pagan/Cook-Weisberg(6)	1167.9[a]	0.0000	1225.4[a]	0.0000	1307.7[a]	0.0000
Multicollinearity						
# VIF scores > 10	5/20		5/19		5/17	
Misspecification						
RESET (7)	43.8[a]	0.0000	16.0[a]	0.0000	16.4[a]	0.0000
Link test(8)	3.06[a]	0.002	1.30[a]	0.194	1.06[a]	0.287
Joint significance tests						
2nd order output terms (df = 3)	137.3	0.0000	129.5	0.0000	130.6	0.0000
Route classes (df = 2)	41.3	0.0000	38.2	0.0000	31.1	0.0000
Provider fixed effects (df = 7)	967.2	0.0000	971.7	0.0000	966.5	0.0000

Note: * Null hypothesis is rejected, as a standard procedure, in these tests, if P-value is less than 0.05. Equivalently, the reported statistic exceeds the critical value for that statistic; † Percentage of n = 7,708 observations; (1) $chi^2(2k)$ where k = 20 for 1st model, and k = 19 for 2nd model and k = 17 for 3rd model. (2) Severe outliers represent about 0.0002% of a normal distribution; (3) Studentized residual > 3; (4) Hat value > $3k/n$; (5) Cook's D > 5 × average Cook's D; (6) $chi^2(1)$; (7) Via powers of the dependent variable, $F(3,n–k–3)$; (8) t-statistic on hat^2; (9) $F(r,n–k–r)$, where r = number of parameters tested, and r = 3 for higher-order output terms, r = 2 for route classes, and r = 8 for provider-specific effects. (10) $chibar^2(1)$; a Approximate, based on OLS regression of $(y - \hat{u}_i)$ on the predictors.

Average Price Effects per Market Segment

Table 4.11 presents the average price predictions using Model 5, actual charges and charges predicted by the DAA 2012 model. These results can be directly compared to those for Model 1 in Table 2.4, because they are averaged over the same set of observations. Table 4.12 shows comparisons of these predictions against the current regulated prices (i.e., the DAA 2012 model). Table 4.13 shows comparisons of these average predicted prices against the actual prices in the dataset.

The inclusion of a dummy variable for services of less than 2.5 Mbps and less than 5 km in Model 5 does not have a substantial effect on the average predicted prices of services of less than 5 Mbps on regulated Metro routes. Models (5b) and (5c) predict these average prices to be $400 and $421 respectively per month which compares to the predictions of Models (1b) and (1c), for the same services, of $395 and $417 per month respectively (Tables 4.11 and 2.5) They are 1 per cent higher than those predicted by Models 1(b) and 1(c).

The model 5 predictions are substantially less than the DAA 2012 predictions for all route categories (Table 4.12).

The model 5 specifications on average predict prices for deregulated routes that range from 8 to 17 per cent less than actual prices for deregulated routes. For metro routes of less than 5Mbps capacity the range is from +3 to -14 per cent of the actual deregulated price (Table 4.13).

These results compare closely with the model 1 results where the overall average ranged from 9 to 18 per cent less than the actual deregulated prices and for low capacity metro services which ranged from 2 per cent higher than actual prices (model 1c) to 15 per cent less than actual prices (model 1a) (Table 2.7).

So the predictions for model 5, for deregulated routes on average are broadly similar to those for model 1. Recall that the model 4 predictions for deregulated routes on average showed a small improvement relative to model 1.

The average prices predicted by Models 5(a), (5b) and (5c) for services of less than 5 Mbps on regulated Metro routes are 24, 13 and 8 per cent below the current regulated price respectively (Table 4.12). The predicted prices are also 6 and 12 per cent above the actual (regulated) prices respectively for models 5(b) and 5(c) but 7 per cent below the actual regulated price for model 5(a) (Table 4.13).

Although the use of the 2.5 Mbps indicator variable results in a lower price for 2 Mbps services on regulated Metro routes, there is an increase in the price for tail-end services. Using Model (5b) the average price for tail-end services is $548 per month, and with Model (5c) it is $575 per month. These predicted prices are higher than the prices predicted by Models (1b) and (1c), namely $483 and $505 respectively. However, they are lower than the predictions of tail-end services for model 4.

Table 4.11: **Model 5: Average actual & predicted prices by market segment ($/month)**

Capacity class	*Route type*	*Actual*	*DAA 2012*	*Model (5a)*	*Model (5b)*	*Models (5c)*
Regulated routes						
Capacity < 5 Mbps	Intercapital	1209	1365	755	849	885
	Metro	378	458	350	400	421
	Regional	1043	782	533	609	653
	All (excl. TEs)	713	635	447	510	543
Capacity 5 - 50 Mbps	Intercapital	2881	6192	1540	1649	1674
	Metro	1074	1676	672	730	773
	Regional	3995	3409	1213	1339	1454
	All (excl. TEs)	2385	2807	970	1059	1128
Capacity 50 - 200 Mbps	Intercapital	6677	13969	2346	2543	2496
	Metro	1524	6405	1327	1425	1504
	Regional	6129	11946	2160	2331	2477
	All (excl. TEs)	3818	9253	1747	1883	1982
Capacity 200+ Mbps	Intercapital	11784	52403	4034	4328	4221
	Metro	2023	13770	1850	1984	2084
	Regional	5198	36426	3266	3533	3699
	All (excl. TEs)	3834	24883	2531	2727	2840
All capacities	Intercapital	4437	12861	1789	1941	1933
	Metro	824	3106	716	785	827
	Regional	2472	6116	1101	1214	1292
	All (excl. TEs)	1718	4877	935	1027	1083
	Tail-end	[redacted]	1101	487	548	575
	All (incl. TEs)		3627	787	869	915

Capacity class	*Route type*	*Actual*	*DAA 2012*	*Model (5a)*	*Model (5b)*	*Models (5c)*
Deregulated						
Capacity < 5 Mbps	Intercapital	812	1340	703	786	814
	Metro	398	440	342	390	409
	Regional	504	653	473	540	566
	All (excl. TEs)	434	516	377	429	449
Capacity 5 - 50 Mbps	Intercapital	1534	4853	1312	1409	1428
	Metro	859	1489	665	724	751
	Regional	1884	4294	1250	1351	1464
	All (excl. TEs)	1010	2161	796	863	892
Capacity 50 - 200 Mbps	Intercapital	2432	13215	2310	2481	2473
	Metro	1399	5202	1235	1329	1384
	Regional	3475	10279	1976	2132	2254
	All (excl. TEs)	1714	6723	1444	1554	1606
Capacity 200+ Mbps	Intercapital	7621	120388	5589	6038	5829
	Metro	1979	14698	1885	2025	2099
	Regional	4449	32605	3135	3403	3541
	All (excl. TEs)	3405	38549	2784	3001	3022
All capacities	Intercapital	3290	38367	2607	2817	2767
	Metro	923	3673	811	884	920
	Regional	2009	9274	1374	1502	1577
	All (excl. TEs)	1307	8433	1080	1175	1203
	Tail-end	.	.	.	.	.
	All (incl. TEs)	1307	8433	1080	1175	1203
Regulated & Deregulated						
Capacity < 5 Mbps	Intercapital	979	1350	725	812	844
	Metro	388	449	346	395	415
	Regional	980	767	526	601	643
	All (excl. TEs)	607	590	420	480	507
Capacity 5 - 50 Mbps	Intercapital	1842	5160	1364	1464	1485
	Metro	911	1535	667	726	757
	Regional	3667	3546	1219	1341	1456
	All (excl. TEs)	1465	2374	853	928	970
Capacity 50 - 200 Mbps	Intercapital	3689	13438	2321	2499	2480
	Metro	1445	5642	1269	1364	1428
	Regional	5645	11642	2126	2295	2436
	All (excl. TEs)	2700	7909	1586	1708	1782
Capacity 200+ Mbps	Intercapital	8311	109128	5331	5755	5563
	Metro	1995	14363	1873	2010	2093
	Regional	4998	35407	3231	3498	3657
	All (excl. TEs)	3583	32878	2679	2887	2947
All capacities	Intercapital	3610	31261	2379	2573	2535
	Metro	882	3439	772	843	881
	Regional	2404	6581	1142	1257	1334
	All (excl. TEs)	1521	6579	1005	1098	1140
	Tail-end	[redacted]	1101	487	548	575
	All (incl. TEs)	[redacted]	5455	898	985	1024

Table 4.12: **Model 5: Difference in predicted prices compared to DAA 2012 (%)**

Capacity class	*Route type*	*Model (5a)*	*Model (5b)*	*Models (5c)*
Regulated routes				
Capacity < 5 Mbps	Intercapital	-45	-38	-35
	Metro	-24	-13	-8
	Regional	-32	-22	-17
	All (excl. TEs)	-30	-20	-14
Capacity 5 - 50 Mbps	Intercapital	-75	-73	-73
	Metro	-60	-56	-54
	Regional	-64	-61	-57
	All (excl. TEs)	-65	-62	-60
Capacity 50 - 200 Mbps	Intercapital	-83	-82	-82
	Metro	-79	-78	-77
	Regional	-82	-80	-79
	All (excl. TEs)	-81	-80	-79
Capacity 200+ Mbps	Intercapital	-92	-92	-92
	Metro	-87	-86	-85
	Regional	-91	-90	-90
	All (excl. TEs)	-90	-89	-89
All capacities	Intercapital	-86	-85	-85
	Metro	-77	-75	-73
	Regional	-82	-80	-79
	All (excl. TEs)	-81	-79	-78
	Tail-end	-56	-50	-48
	All (incl. TEs)	-78	-76	-75
Deregulated				
Capacity < 5 Mbps	Intercapital	-48	-41	-39
	Metro	-22	-11	-7
	Regional	-28	-17	-13
	All (excl. TEs)	-27	-17	-13
Capacity 5 - 50 Mbps	Intercapital	-73	-71	-71
	Metro	-55	-51	-50
	Regional	-71	-69	-66
	All (excl. TEs)	-63	-60	-59
Capacity 50 - 200 Mbps	Intercapital	-83	-81	-81
	Metro	-76	-74	-73
	Regional	-81	-79	-78
	All (excl. TEs)	-79	-77	-76
Capacity 200+ Mbps	Intercapital	-95	-95	-95
	Metro	-87	-86	-86
	Regional	-90	-90	-89
	All (excl. TEs)	-93	-92	-92
All capacities	Intercapital	-93	-93	-93
	Metro	-78	-76	-75
	Regional	-85	-84	-83
	All (excl. TEs)	-87	-86	-86
	Tail-end	.	.	.
	All (incl. TEs)	-87	-86	-86

Capacity class	*Route type*	*Model (5a)*	*Model (5b)*	*Models (5c)*
Regulated & Deregulated				
Capacity < 5 Mbps	Intercapital	-46	-40	-38
	Metro	-23	-12	-7
	Regional	-31	-22	-16
	All (excl. TEs)	-29	-19	-14
Capacity 5 - 50 Mbps	Intercapital	-74	-72	-71
	Metro	-57	-53	-51
	Regional	-66	-62	-59
	All (excl. TEs)	-64	-61	-59
Capacity 50 - 200 Mbps	Intercapital	-83	-81	-82
	Metro	-78	-76	-75
	Regional	-82	-80	-79
	All (excl. TEs)	-80	-78	-77
Capacity 200+ Mbps	Intercapital	-95	-95	-95
	Metro	-87	-86	-85
	Regional	-91	-90	-90
	All (excl. TEs)	-92	-91	-91
All capacities	Intercapital	-92	-92	-92
	Metro	-78	-75	-74
	Regional	-83	-81	-80
	All (excl. TEs)	-85	-83	-83
	Tail-end	-56	-50	-48
	All (incl. TEs)	-84	-82	-81

Table 4.13: **Model 5: Difference in predicted prices compared to Actual (%)**

Capacity class	*Route type*	*DAA 2012*	*Model (5a)*	*Model (5b)*	*Models (5c)*
Regulated routes					
Capacity < 5 Mbps	Intercapital	13	-38	-30	-27
	Metro	21	-7	6	12
	Regional	-25	-49	-42	-37
	All (excl. TEs)	-11	-37	-28	-24
Capacity 5 - 50 Mbps	Intercapital	115	-47	-43	-42
	Metro	56	-37	-32	-28
	Regional	-15	-70	-66	-64
	All (excl. TEs)	18	-59	-56	-53
Capacity 50 - 200 Mbps	Intercapital	109	-65	-62	-63
	Metro	320	-13	-7	-1
	Regional	95	-65	-62	-60
	All (excl. TEs)	142	-54	-51	-48
Capacity 200+ Mbps	Intercapital	345	-66	-63	-64
	Metro	581	-9	-2	3
	Regional	601	-37	-32	-29
	All (excl. TEs)	549	-34	-29	-26
All capacities	Intercapital	190	-60	-56	-56
	Metro	277	-13	-5	0
	Regional	147	-55	-51	-48
	All (excl. TEs)	184	-46	-40	-37

Capacity class	*Route type*	*DAA 2012*	*Model (5a)*	*Model (5b)*	*Models (5c)*
	Tail-end	166	18	32	39
	All (incl. TEs)	182	-39	-32	-29
Deregulated					
Capacity < 5 Mbps	Intercapital	65	-13	-3	0
	Metro	10	-14	-2	3
	Regional	29	-6	7	12
	All (excl. TEs)	19	-13	-1	3
Capacity 5 - 50 Mbps	Intercapital	216	-14	-8	-7
	Metro	73	-23	-16	-13
	Regional	128	-34	-28	-22
	All (excl. TEs)	114	-21	-15	-12
Capacity 50 - 200 Mbps	Intercapital	443	-5	2	2
	Metro	272	-12	-5	-1
	Regional	196	-43	-39	-35
	All (excl. TEs)	292	-16	-9	-6
Capacity 200+ Mbps	Intercapital	1480	-27	-21	-24
	Metro	643	-5	2	6
	Regional	633	-30	-24	-20
	All (excl. TEs)	1032	-18	-12	-11
All capacities	Intercapital	1066	-21	-14	-16
	Metro	298	-12	-4	0
	Regional	362	-32	-25	-22
	All (excl. TEs)	545	-17	-10	-8
	Tail-end	.	.	.	.
	All (incl. TEs)	545	-17	-10	-8
Regulated & Deregulated					
Capacity < 5 Mbps	Intercapital	38	-26	-17	-14
	Metro	16	-11	2	7
	Regional	-22	-46	-39	-34
	All (excl. TEs)	-3	-31	-21	-16
Capacity 5 - 50 Mbps	Intercapital	180	-26	-21	-19
	Metro	68	-27	-20	-17
	Regional	-3	-67	-63	-60
	All (excl. TEs)	62	-42	-37	-34
Capacity 50 - 200 Mbps	Intercapital	264	-37	-32	-33
	Metro	291	-12	-6	-1
	Regional	106	-62	-59	-57
	All (excl. TEs)	193	-41	-37	-34
Capacity 200+ Mbps	Intercapital	1213	-36	-31	-33
	Metro	620	-6	1	5
	Regional	608	-35	-30	-27
	All (excl. TEs)	818	-25	-19	-18
All capacities	Intercapital	766	-34	-29	-30
	Metro	290	-13	-4	0
	Regional	174	-53	-48	-45
	All (excl. TEs)	332	-34	-28	-25
	Tail-end	166	18	32	39
	All (incl. TEs)	322	-31	-24	-21

Cost elasticities of capacity and distance

Table 4.14 shows the cost elasticities with respect to capacity and distance. This information is useful given the change in the sign of the coefficient on log distance. It shows that the marginal effect of distance on cost remains positive over the range of values for capacity and distance selected.

Table 4.14: **Model 5: Cost elasticities**

	Model (5a)		Model (5b)		Model (5c)	
	capacity	*distance*	*capacity*	*distance*	*Capacity*	*distance*
Deregulated routes						
10th	0.411	-0.003	0.404	-0.006	0.401	0.016
50th	0.352	0.067	0.348	0.065	0.343	0.085
90th	0.276	0.206	0.274	0.206	0.265	0.227
Regulated routes						
10th	0.411	0.027	0.405	0.024	0.400	0.046
50th	0.381	0.091	0.376	0.089	0.369	0.112
90th	0.294	0.185	0.291	0.185	0.283	0.206

Source: Economic Insights.

A shortcoming of introducing a specific effect for 2.5 Mbps services is that it introduces a discontinuity into the price formula. For example, consider a service on a 4 km Metro route with an Ethernet interface. Using Model (5c) the predicted monthly charge for a service with a capacity of 2.499 Mbps will be $341 per month. However, for a service of 2.500 Mbps, the predicted price is $431 per month. Theoretically, this may be a problem, however, in reality contracts are sold at discrete capacity intervals. For instance, the next capacity interval up from 2.5 Mbps with more than just a handful of services is a 10 Mbps service by which point the shock of the price shift has worn off.

4.3 Piecewise Regression

One method of dealing with systematic differences in the pricing of low capacity services while avoiding a discontinuity in the price formula is piecewise regression. This method provides for a change in the slope of a function at a particular point, referred to as a "knot".

The piecewise regression model in which there is a single known knot for variable $\ln y_1$ can be formulated as follows. Equation (4.1) is equivalent to equation (2.1), but all terms that do not involve the variable $\ln y_1$ are grouped together into the term Ω for convenience.

$$\ln C = \gamma_0 + \gamma_1 \ln y_1 + \frac{1}{2}\gamma_{11}\left(\ln y_1\right)^2 + \gamma_{12} \ln y_1 \ln y_2 + \Omega \tag{4.1}$$

The piecewise model this can be represented as:

$$(4.2) \qquad \ln C = \gamma_{0a} + \gamma_{1a}\ln y_1 + \frac{1}{2}\gamma_{11a}\left(\ln y_1\right)^2 + \gamma_{12a}\ln y_1 \ln y_2 + \left[\gamma_{0b} + \gamma_{1b}\ln y_1 + \frac{1}{2}\gamma_{11b}\left(\ln y_1\right)^2 + \gamma_{12b}\ln y_1 \ln y_2\right] D + \Omega$$

Where: $D = 1$ when $\ln y_1 \geq k$, and $D = 0$ when $\ln y_1 < k$, and k is the known value of $\ln y_1$ at the knot. The requirement that the lines meet at the knot means that the expression in square brackets must be equal to zero when $\ln y_1 = k$. Therefore:

$$(4.3) \qquad \gamma_{0b} = -\gamma_{1b}k - \frac{1}{2}\gamma_{11b}k^2 - \gamma_{12b}k\ln y_2$$

This expression for γ_{0b} can be substituted into equation (4.2) to produce:

$$(4.4) \qquad \ln C = \gamma_{0a} + \gamma_{1a}\ln y_1 + \frac{1}{2}\gamma_{11a}\left(\ln y_1\right)^2 + \gamma_{12a}\ln y_1 \ln y_2 + \left[\gamma_{1b}\left(\ln y_1 - k\right) + \frac{1}{2}\gamma_{11b}\left(\left(\ln y_1\right)^2 - k^2\right) + \gamma_{12b}\left(\ln y_1 - k\right)\ln y_2\right] D + \Omega$$

Equation (4.4) is the model presented in this section.

Econometric Results

Table 4.15 shows the results of estimating the piecewise regression model described. The sign of the coefficient on log capacity is negative below 2.5 Mbps, but positive for higher capacities (indicated by the sum of the coefficient on log capacity and the coefficient on Δlog capacity (which is the positive difference from the knot, *k*). The coefficient on squared log capacity is positive throughout.

Table 4.16 shows goodness-of-fit statistics. The fit is comparable to Model 1. Table 4.17 shows the results of diagnostic tests. Unsurprisingly there is a higher degree of multicollinearity because the outputs each appear in several variables.

Elasticities

The elasticities of cost to capacity and distance are shown in Table 4.18. In some of the models, when capacity is less than 2.5 Mbps, the elasticities of cost to capacity can be negative. This is less than ideal because it implies that price decreases with higher capacity, although this occurs only over a short range. This greatly reduces the utility of this piecewise regression model. Options that could be tested to possibly remedy this include: (a) constraining the elasticity of cost to capacity to be equal to zero when capacity is less than 2.5 Mbps; or (b) testing a more complicated model where there are knots for both capacity and distance. However, these elaborations are not explored in this study.

Table 4.15: **Model 6: Piecewise RE models (2015 data)***

	Model (6a)		Model (6b)		Model (6c)	
Predictor	*Coeff.*	*t-stat.*	*Coeff.*	*t-stat.*	*Coeff.*	*t-stat.*
constant	6.23957	1.30	12.39297	2.59	12.24275	2.56
log capacity	-1.70430	-0.14	-20.15452	-1.66	-18.96859	-1.56
log distance	0.23730	6.23	0.21854	5.70	0.25014	6.54
0.5(log capacity)^2	4.12544	0.27	26.84087	1.78	25.42676	1.69
0.5(log distance)^2	0.02556	4.24	0.02629	4.31	0.02703	4.36
(log capacity)(log distance)	-0.24786	-6.01	-0.23075	-5.57	-0.24066	-5.81
Δlog capacity**	2.10923	0.17	20.56122	1.70	19.37060	1.60
Δ0.5(log capacity)^2**	-4.14938	-0.27	-26.86527	-1.78	-25.45032	-1.69
Δ(log capacity)(log distance)**	0.24973	5.89	0.23235	5.45	0.24086	5.65
log route t'put	-0.02585	-5.09	-0.02859	-5.58	.	.
log ESA t'put	0.05361	6.13	0.05049	5.72	.	.
contract start date	-0.00007	-9.88	.	.	.	.
route class 2 (Metro)	0.09874	1.45	0.09461	1.37	0.15369	2.22
route class 3 (Regional)	0.25185	4.62	0.24279	4.42	0.25610	4.59
Provider #1						
Provider #3						
Provider #4						
Provider #5						
Provider #6						
Provider #7						
Provider #8						
Provider #9						
interface-type 3 (SDH)	0.26820	12.96	0.32180	16.02	0.31979	15.89
σ(u)	0.29379		0.29809		0.30792	
σ(e)	0.41684		0.41901		0.41843	

Source: Economic Insights estimation results.

Notes: * Known knot at 2.5 Mbps. ** Additive effect for the increment above the knot.

Table 4.16: **Model 6: Goodness-of-fit**

	Model (6a)		Model (6b)		Model (6c)	
Goodness-of-fit statistic	*10-fold validation (avg)*	*Full sample*	*10-fold validation (avg)*	*Full sample*	*10-fold validation (avg)*	*Full sample*
obs	6,937	7,708	6,937	7,708	6,937	7,708
Within-sample						
R^2*	0.68758	0.68707	0.68393	0.68338	0.67679	0.67636
BIC	.	10134.5	.	10222.6	.	10246.2
RMSE (based on ue)	0.50356	0.50407	0.50643	0.50696	0.51283	0.51334
MAE (based on ue)	0.35886	0.35939	0.36043	0.36104	0.36937	0.36975
Out-of-sample						
R^2*	0.68535	.	0.68168	.	0.67473	.
RMSE (based on ue)	0.50534	.	0.50815	.	0.51434	.
MAE (based on ue)	0.36012	.	0.36166	.	0.37051	.

Notes: * Squared correlation between fitted and actual dependent.

Table 4.17: **Model 6: Statistical tests**

	Model (6a)		Model (6b)		Model (6c)	
	Stat.	*P-value**	*Stat.*	*P-value**	*Stat.*	*P-value**
Normality of residuals						
Doornik-Hansen(1)	4863.3	0.0000	4979.9	0.0000	4893.9	0.0000
IQR (% severe outliers) (2)†	2.19%		2.61%		2.58%	
Homoscedasticity						
Breusch-Pagan/Cook-Weisberg(6)	1161.4[a]	0.0000	1216.0[a]	0.0000	1303.5[a]	0.0000
Multicollinearity						
# VIF scores > 10	9/22		9/21		9/19	
Misspecification						
RESET (7)	44.0[a]	0.0000	16.3[a]	0.0000	16.5[a]	0.0000
Link test(8)	3.05[a]	0.002	1.23[a]	0.219	0.95[a]	0.340
Joint significance tests						
2nd order output terms (df = 3)	51.7	0.0000	50.5	0.0000	53.1	0.0000
Incremental effects (df = 3)**	62.0	0.0000	51.8	0.0000	55.9	0.0000
Route classes (df = 2)	38.0	0.0000	34.7	0.0000	28.6	0.0000
Provider fixed effects (df = 7)	900.6	0.0000	908.4	0.0000	902.4	0.0000

Note: * Null hypothesis is rejected, as a standard procedure, in these tests, if P-value is less than 0.05. Equivalently, the reported statistic exceeds the critical value for that statistic; ** Additive effects for incremental capacity above the knot. † Percentage of n = 7,708 observations; (1) $chi^2(2k)$ where k = 22 for 1st model, and k = 21 for 2nd model and k = 19 for 3rd model. (2) Severe outliers represent about 0.0002% of a normal distribution; (3) Studentized residual > 3; (4) Hat value > $3k/n$; (5) Cook's D > 5 × average Cook's D; (6) $chi^2(1)$; (7) Via powers of the dependent variable, $F(3,n–k–3)$; (8) t-statistic on hat^2; (9) $F(r,n–k–r)$, where r = number of parameters tested, and r = 3 for higher-order output terms, r = 2 for route classes, and r = 8 for provider-specific effects. (10) $chibar^2(1)$; a Approximate, based on OLS regression of $(y - \hat{u}_i)$ on the predictors.

Table 4.18: **Model 6: Cost elasticities**

	Model (6a)		Model (6b)		Model (6c)	
	capacity	*distance*	*capacity*	*distance*	*capacity*	*distance*
Deregulated routes						
10th	1.021	0.079	-1.675	0.073	-1.475	0.098
50th	0.341	0.307	0.341	0.290	0.335	0.319
90th	0.281	0.416	0.278	0.400	0.269	0.429
Regulated routes						
10th	0.809	0.101	-1.872	0.095	-1.680	0.121
50th	0.368	0.324	0.368	0.307	0.360	0.339
90th	0.295	0.399	0.293	0.383	0.284	0.412

Source: Economic Insights.

Average Price Effects per Market Segment

The average predicted prices for the model 6 specifications are shown in Table 4.19 and the percentage differences from the charges predicted by the 2012 model are shown in Table 4.20.

The average predicted price on all regulated routes in Model 6 are lower than those for Model 1. For Model (6b) it is $851 per month and for Model (6c) it is $897 per month, which are far lower than the predicted overall prices from Model (1b) of $848 per month, and from Model (1c) of $895 per month (tables 4.19 and 2.5).

The model 6 predictions are substantially less than the DAA 2012 predictions for all route categories (Table 4.20). The predicted prices for models 6(b) and 6(c) are 34 and 31 per cent lower respectively for low capacity services on regulated Metro routes than the predictions of the 2012 model.

The model 6 specifications on average predict prices for deregulated routes that range from 8 to 17 per cent less than actual prices for deregulated routes. For metro routes of less than 5Mbps capacity the range is from 2 per cent higher to 15 per cent lower than the actual deregulated price (Table 4.20).

These results compare with the model 1 results where the overall average ranged from 9 to 18 per cent less than the actual deregulated prices and for low capacity metro services which ranged from 2 per cent higher than actual prices (model 1c) to 15 per cent less than actual prices (model 1a) (Table 2.7).

So the predictions for model 6, for deregulated routes on average are broadly similar to those for model 1. Recall that the model 4 predictions for deregulated routes on average showed a small improvement relative to model 1 and the model 5 predictions were broadly similar to those for model 1.

Turning to the results for regulated routes, in Model (6b) the predicted price of $395 per month for regulated Metro services of less than 5 Mbps is 5 per cent higher than the actual price of $378, and the Model (6c) average price is 10 per cent higher than actual prices. These predictions are nearly identical to the predictions for model 1.

In Model 6 the predicted prices for tail-end services are also higher than the actual prices. In Model 6(b) the average predicted price for tail-end routes is $478 per month compared to the actual price of [redacted], and using Model (6c) the predicted price is $499 per month. These predicted prices are similar to the prices predicted by Models (1b) and (1c), namely $483 and $505 respectively and lower than the predictions of tail end services for models 4 and 5.

Table 4.19: **Model 6: Predicted prices by market segment ($/month)**

Capacity class	*Route type*	*Actual*	*DAA 2012*	*Model (6a)*	*Model (6b)*	*Models (6c)*
Regulated routes						
Capacity < 5 Mbps	Intercapital	1209	1365	819	903	947
	Metro	378	458	343	395	416
	Regional	1043	782	547	626	674
	All (excl. TEs)	713	635	452	517	552
Capacity 5 - 50 Mbps	Intercapital	2881	6192	1490	1592	1616
	Metro	1074	1676	695	748	795
	Regional	3995	3409	1223	1342	1460
	All (excl. TEs)	2385	2807	980	1063	1136
Capacity 50 - 200 Mbps	Intercapital	6677	13969	2310	2507	2457
	Metro	1524	6405	1354	1449	1533
	Regional	6129	11946	2166	2332	2481
	All (excl. TEs)	3818	9253	1761	1893	1996
Capacity 200+ Mbps	Intercapital	11784	52403	3974	4253	4141
	Metro	2023	13770	1885	2016	2122
	Regional	5198	36426	3304	3557	3729
	All (excl. TEs)	3834	24883	2562	2749	2868
All capacities	Intercapital	4437	12861	1786	1929	1923
	Metro	824	3106	723	791	835
	Regional	2472	6116	1116	1228	1310
	All (excl. TEs)	1718	4877	945	1036	1095
	Tail-end	[redacted]	1101	422	478	499
	All (incl. TEs)	[redacted]	3627	772	851	897
Deregulated						
Capacity < 5 Mbps	Intercapital	812	1340	762	840	874
	Metro	398	440	337	386	404
	Regional	504	653	475	544	572
	All (excl. TEs)	434	516	377	429	450
Capacity 5 - 50 Mbps	Intercapital	1534	4853	1280	1370	1389
	Metro	859	1489	674	727	756
	Regional	1884	4294	1250	1343	1459
	All (excl. TEs)	1010	2161	797	858	889
Capacity 50 - 200 Mbps	Intercapital	2432	13215	2247	2413	2402
	Metro	1399	5202	1244	1334	1392
	Regional	3475	10279	1987	2137	2263
	All (excl. TEs)	1714	6723	1443	1549	1603
Capacity 200+ Mbps	Intercapital	7621	120388	5605	6011	5798
	Metro	1979	14698	1892	2026	2105
	Regional	4449	32605	3184	3442	3587
	All (excl. TEs)	3405	38549	2797	3000	3024
All capacities	Intercapital	3290	38367	2600	2794	2744
	Metro	923	3673	813	883	921
	Regional	2009	9274	1387	1512	1591
	All (excl. TEs)	1307	8433	1082	1172	1203
	Tail-end	.	.	.	.	.
	All (incl. TEs)	1307	8433	1082	1172	1203

Capacity class	*Route type*	*Actual*	*DAA 2012*	*Model (6a)*	*Model (6b)*	*Models (6c)*
Regulated & Deregulated						
Capacity < 5 Mbps	Intercapital	979	1350	786	866	905
	Metro	388	449	340	390	410
	Regional	980	767	538	616	662
	All (excl. TEs)	607	590	423	484	513
Capacity 5 - 50 Mbps	Intercapital	1842	5160	1328	1421	1441
	Metro	911	1535	679	732	766
	Regional	3667	3546	1227	1342	1460
	All (excl. TEs)	1465	2374	858	926	970
Capacity 50 - 200 Mbps	Intercapital	3689	13438	2265	2440	2418
	Metro	1445	5642	1284	1376	1443
	Regional	5645	11642	2133	2296	2441
	All (excl. TEs)	2700	7909	1592	1710	1787
Capacity 200+ Mbps	Intercapital	8311	109128	5335	5720	5524
	Metro	1995	14363	1890	2023	2111
	Regional	4998	35407	3272	3527	3691
	All (excl. TEs)	3583	32878	2699	2896	2960
All capacities	Intercapital	3610	31261	2373	2553	2516
	Metro	882	3439	776	845	885
	Regional	2404	6581	1156	1270	1351
	All (excl. TEs)	1521	6579	1011	1101	1146
	Tail-end	[redacted]	1101	422	478	499
	All (incl. TEs)	[redacted]	5455	890	973	1013

Table 4.20: **Model 6: Difference in predicted prices compared to DAA 2012 (%)**

Capacity class	*Route type*	*DAA 2015*	*Model (6a)*	*Model (6b)*	*Models (6c)*
Regulated routes					
Capacity < 5 Mbps	Intercapital	-31	-40	-34	-31
	Metro	-8	-25	-14	-9
	Regional	-26	-30	-20	-14
	All (excl. TEs)	-20	-29	-19	-13
Capacity 5 - 50 Mbps	Intercapital	-67	-76	-74	-74
	Metro	-46	-59	-55	-53
	Regional	-62	-64	-61	-57
	All (excl. TEs)	-58	-65	-62	-60
Capacity 50 - 200 Mbps	Intercapital	-78	-83	-82	-82
	Metro	-73	-79	-77	-76
	Regional	-79	-82	-80	-79
	All (excl. TEs)	-77	-81	-80	-78
Capacity 200+ Mbps	Intercapital	-89	-92	-92	-92
	Metro	-81	-86	-85	-85
	Regional	-88	-91	-90	-90
	All (excl. TEs)	-86	-90	-89	-88
All capacities	Intercapital	-81	-86	-85	-85
	Metro	-70	-77	-75	-73
	Regional	-79	-82	-80	-79
	All (excl. TEs)	-76	-81	-79	-78

Capacity class	*Route type*	*DAA 2015*	*Model (6a)*	*Model (6b)*	*Models (6c)*
	Tail-end	-41	-62	-57	-55
	All (incl. TEs)	-73	-79	-77	-75
Deregulated					
Capacity < 5 Mbps	Intercapital	-30	-43	-37	-35
	Metro	-3	-23	-12	-8
	Regional	-16	-27	-17	-12
	All (excl. TEs)	-9	-27	-17	-13
Capacity 5 - 50 Mbps	Intercapital	-64	-74	-72	-71
	Metro	-45	-55	-51	-49
	Regional	-67	-71	-69	-66
	All (excl. TEs)	-54	-63	-60	-59
Capacity 50 - 200 Mbps	Intercapital	-77	-83	-82	-82
	Metro	-70	-76	-74	-73
	Regional	-77	-81	-79	-78
	All (excl. TEs)	-73	-79	-77	-76
Capacity 200+ Mbps	Intercapital	-92	-95	-95	-95
	Metro	-82	-87	-86	-86
	Regional	-88	-90	-89	-89
	All (excl. TEs)	-89	-93	-92	-92
All capacities	Intercapital	-90	-93	-93	-93
	Metro	-72	-78	-76	-75
	Regional	-82	-85	-84	-83
	All (excl. TEs)	-83	-87	-86	-86
	Tail-end	.	.	.	.
	All (incl. TEs)	-83	-87	-86	-86
Regulated & Deregulated					
Capacity < 5 Mbps	Intercapital	-30	-42	-36	-33
	Metro	-6	-24	-13	-9
	Regional	-25	-30	-20	-14
	All (excl. TEs)	-16	-28	-18	-13
Capacity 5 - 50 Mbps	Intercapital	-65	-74	-72	-72
	Metro	-46	-56	-52	-50
	Regional	-63	-65	-62	-59
	All (excl. TEs)	-55	-64	-61	-59
Capacity 50 - 200 Mbps	Intercapital	-78	-83	-82	-82
	Metro	-72	-77	-76	-74
	Regional	-79	-82	-80	-79
	All (excl. TEs)	-75	-80	-78	-77
Capacity 200+ Mbps	Intercapital	-92	-95	-95	-95
	Metro	-82	-87	-86	-85
	Regional	-88	-91	-90	-90
	All (excl. TEs)	-88	-92	-91	-91
All capacities	Intercapital	-89	-92	-92	-92
	Metro	-71	-77	-75	-74
	Regional	-80	-82	-81	-79
	All (excl. TEs)	-80	-85	-83	-83
	Tail-end	-41	-62	-57	-55
	All (incl. TEs)	-79	-84	-82	-81

Table 4.21: **Model 6: Difference in predicted prices compared to Actual (%)**

Capacity class	*Route type*	*DAA 2012*	*Model (6a)*	*Model (6b)*	*Models (6c)*
Regulated routes					
Capacity < 5 Mbps	Intercapital	13	-32	-25	-22
	Metro	21	-9	5	10
	Regional	-25	-48	-40	-35
	All (excl. TEs)	-11	-37	-27	-23
Capacity 5 - 50 Mbps	Intercapital	115	-48	-45	-44
	Metro	56	-35	-30	-26
	Regional	-15	-69	-66	-63
	All (excl. TEs)	18	-59	-55	-52
Capacity 50 - 200 Mbps	Intercapital	109	-65	-62	-63
	Metro	320	-11	-5	1
	Regional	95	-65	-62	-60
	All (excl. TEs)	142	-54	-50	-48
Capacity 200+ Mbps	Intercapital	345	-66	-64	-65
	Metro	581	-7	0	5
	Regional	601	-36	-32	-28
	All (excl. TEs)	549	-33	-28	-25
All capacities	Intercapital	190	-60	-57	-57
	Metro	277	-12	-4	1
	Regional	147	-55	-50	-47
	All (excl. TEs)	184	-45	-40	-36
	Tail-end	166	2	15	21
	All (incl. TEs)	182	-40	-34	-30
Deregulated					
Capacity < 5 Mbps	Intercapital	65	-6	3	8
	Metro	10	-15	-3	2
	Regional	29	-6	8	13
	All (excl. TEs)	19	-13	-1	4
Capacity 5 - 50 Mbps	Intercapital	216	-17	-11	-9
	Metro	73	-22	-15	-12
	Regional	128	-34	-29	-23
	All (excl. TEs)	114	-21	-15	-12
Capacity 50 - 200 Mbps	Intercapital	443	-8	-1	-1
	Metro	272	-11	-5	-1
	Regional	196	-43	-38	-35
	All (excl. TEs)	292	-16	-10	-6
Capacity 200+ Mbps	Intercapital	1480	-26	-21	-24
	Metro	643	-4	2	6
	Regional	633	-28	-23	-19
	All (excl. TEs)	1032	-18	-12	-11
All capacities	Intercapital	1066	-21	-15	-17
	Metro	298	-12	-4	0
	Regional	362	-31	-25	-21
	All (excl. TEs)	545	-17	-10	-8
	Tail-end	.	.	.	.
	All (incl. TEs)	545	-17	-10	-8

Capacity class	*Route type*	*DAA 2012*	*Model (6a)*	*Model (6b)*	*Models (6c)*
Regulated & Deregulated					
Capacity < 5 Mbps	Intercapital	38	-20	-11	-8
	Metro	16	-12	1	6
	Regional	-22	-45	-37	-32
	All (excl. TEs)	-3	-30	-20	-15
Capacity 5 - 50 Mbps	Intercapital	180	-28	-23	-22
	Metro	68	-25	-20	-16
	Regional	-3	-67	-63	-60
	All (excl. TEs)	62	-41	-37	-34
Capacity 50 - 200 Mbps	Intercapital	264	-39	-34	-34
	Metro	291	-11	-5	0
	Regional	106	-62	-59	-57
	All (excl. TEs)	193	-41	-37	-34
Capacity 200+ Mbps	Intercapital	1213	-36	-31	-34
	Metro	620	-5	1	6
	Regional	608	-35	-29	-26
	All (excl. TEs)	818	-25	-19	-17
All capacities	Intercapital	766	-34	-29	-30
	Metro	290	-12	-4	0
	Regional	174	-52	-47	-44
	All (excl. TEs)	332	-34	-28	-25
	Tail-end	166	2	15	21
	All (incl. TEs)	322	-31	-25	-22

5 STOCHASTIC FRONTIER ANALYSIS

In the stochastic frontier model[13] the stochastic term in equations (2.1), namely ε_{it}, is composed of the two elements: $\varepsilon_{it} = u_i + e_{it}$, but unlike the random effects model, where u_i is a normally distributed random variable with zero mean, in the stochastic frontier model u_i is a strictly positive random variable (in the cost function application) which therefore has a positive mean value. There are a wide variety of methods of specifying the term u_i, in terms of either the chosen stochastic distribution, or whether the mean or variance of u_i, are functions of time or of other variables. However, as the methods increase in complexity the difficulty of successfully estimating the model also usually increases. Here we use one of the simplest and most commonly used specifications for the distribution of u_i, the half-normal distribution. This is called the Normal-Half Normal stochastic frontier specification.

The random effects model assumes that for each route there is a positive or negative disturbance to the cost function intercept, and these effects are normally distributed with zero mean. Whereas, the SFA model assumes that for each route there may be an additional positive disturbance to the cost function intercept but not a negative disturbance, and these positive disturbances are typically assumed to have either a half-normal or truncated-normal distribution, and do not have a zero mean. The additional positive disturbance is typically interpreted (in a cost function) as reflecting estimated cost inefficiencies based on observed asymmetries in residuals which tend to be related to higher cost observations. As a result an efficient cost frontier is estimated that generally produces lower forecast costs for efficient firms than a cost function that is estimated with standard assumptions about residuals.

Econometric Results

Table 5.1 shows the results of estimating the three specifications presented in Table 2.1, but using the stochastic frontier method instead of the random effects method. Table 5.1 also includes some goodness-of-fit measures. Fewer goodness-of-fit measures are reported here because, previously we have calculated measures such as RMSE and MAE based on the *e*'s (i.e., including both the *u*'s and the *v*'s) but because of the skewed shape and non-zero mean of the *u*'s in this model, comparable measures of fit would not be meaningful.

[13] For example, see Section 19.2.4 in Greene (2012).

Table 5.1: **Model 7: Stochastic frontier (Normal-Half Normal) (2014 data)**

	Model (7a)		Model (7b)		Model (7c)	
Predictor	*Coeff.*	*t-stat.*	*Coeff.*	*t-stat.*	*Coeff.*	*t-stat.*
constant	5.00017	29.90	4.09880	35.17	4.55207	52.82
log capacity	0.50498	44.86	0.49694	44.23	0.49674	43.86
log distance	-0.00308	-0.13	-0.00794	-0.33	0.01834	0.77
0.5(log capacity)^2	-0.04079	-14.41	-0.03964	-13.99	-0.03960	-13.84
0.5(log distance)^2	0.03222	4.86	0.03280	4.92	0.03716	5.58
(log capacity)(log distance)	-0.00478	-3.23	-0.00479	-3.23	-0.00850	-5.84
log route t'put	-0.05659	-10.41	-0.05915	-10.86	.	.
log ESA t'put	0.07849	8.80	0.07589	8.54	.	.
contract start date	-0.00005	-7.54	.	.	.	.
route class 2 (Metro)	0.18980	2.52	0.18605	2.46	0.27856	3.70
route class 3 (Regional)	0.20245	3.44	0.20371	3.44	0.22842	3.81
Provider #1						
Provider #3						
Provider #4						
Provider #5						
Provider #6						
Provider #7						
Provider #8						
Provider #9						
interface-type 3 (SDH)	0.18592	9.75	0.22737	12.42	0.21575	11.67
σ(u)	0.33315		0.33881		0.35564	
σ(e)	0.18009		0.18105		0.18477	
Goodness-of-fit						
R^2*	0.68380		0.68024		0.67721	
BIC	10,524.0		10,571.7		10,689.1	

Source: Economic Insights estimation results.

Notes: * Squared correlation between fitted and actual dependent.

Table 5.2 shows the elasticities of charges with respect to capacity and distance. Here the elasticities are all positive, as required, and the magnitudes of the elasticities are consistent with our broad expectations.

Table 5.2: **Model 7: Cost elasticities**

	Model (6a)		Model (6b)		Model (6c)	
	capacity	*distance*	*capacity*	*distance*	*capacity*	*distance*
Deregulated routes						
10th	0.474	0.011	0.467	0.007	0.465	0.033
50th	0.376	0.065	0.372	0.061	0.362	0.088
90th	0.241	0.181	0.239	0.180	0.215	0.214
Regulated routes						
10th	0.470	0.039	0.463	0.035	0.457	0.064
50th	0.416	0.093	0.410	0.090	0.397	0.124
90th	0.271	0.164	0.269	0.163	0.246	0.196

Source: Economic Insights.

Table 5.3 shows the results of the diagnostic tests, and again the results are broadly similar to those obtained with the original data sample with the exception that there are fewer severe outliers.

Table 5.3: **Model 7: Statistical tests**

	Model (7a)		Model (7b)		Model (7c)	
	Stat.	*P-value**	*Stat.*	*P-value**	*Stat.*	*P-value**
Normality of residuals						
Doornik-Hansen(1)	2514.0	0.0000	2678.3	0.0000	2375.1	0.0000
IQR (% severe outliers) (2)†	0.59%		0.63%		0.58%	
Homoscedasticity						
Breusch-Pagan/Cook-Weisberg(6)	903.0[a]	0.0000	876.5[a]	0.0000	1064.0[a]	0.0000
Multicollinearity						
# VIF scores > 10	5/19		5/18		5/16	
Misspecification						
RESET (7)	26.0[a]	0.0000	11.2[a]	0.0000	11.2[a]	0.0000
Link test(8)	3.55[a]	0.000	2.17[a]	0.030	1.39[a]	0.165
Joint significance tests						
2nd order output terms (df = 3)	292.2	0.0000	278.7	0.0000	332.5	0.0000
Route classes (df = 2)	12.2	0.0022	12.4	0.0020	15.4	0.0005
Provider fixed effects (df = 7)	938.7	0.0000	936.9	0.0000	929.7	0.0000

Note: * Null hypothesis is rejected, as a standard procedure, in these tests, if P-value is less than 0.05. Equivalently, the reported statistic exceeds the critical value for that statistic; † Percentage of n = 7,708 observations; (1) $chi^2(2k)$ where k = 19 for 1st model, and k = 18 for 2nd model and k = 16 for 3rd model. (2) Severe outliers represent about 0.0002% of a normal distribution; (3) Studentized residual > 3; (4) Hat value > $3k/n$; (5) Cook's D > 5 × average Cook's D; (6) $chi^2(1)$; (7) Via powers of the dependent variable, $F(3,n–k–3)$; (8) t-statistic on hat^2; (9) $F(r,n–k–r)$, where r = number of parameters tested, and r = 3 for higher-order output terms, r = 2 for route classes, and r = 8 for provider-specific effects. (10) $chibar^2(1)$; a Approximate, based on OLS regression of $(y - \hat{u}_i)$ on the predictors.

Average Price Effects per Market Segment

Table 5.4 compares the price predictions using Model 7 against actual charges and against the DAA 2012 model, which corresponds to the regulated price formula. The overall average predicted price on all regulated routes using Model (7c) is $523 per month, which is more than 40 per cent lower than the average predicted price on all regulated routes obtained using Model (1c), namely $895 per month.

Table 5.5 presents the percentage changes from the prices predicted by the 2012 model. Over all regulated routes, the predicted prices of Model (7c) would represent an 86 per cent decrease compared to the 2014 actual prices.

Table 5.6 presents the percentage changes from the actual price implied by the predicted prices. Over all the deregulated routes, the predicted prices range from 44 to 49 per cent less than the actual prices. Over all regulated routes, the predicted prices of Model (7c) would represent a 59 per cent decrease compared to the 2014 actual prices. This compares to the 30 per cent reduction implied by Model (1c).

These findings support our previous contention that the stochastic frontier model would predict lower prices than the random effects model. We also note that relative to the random effects model the unexplained variation in the data is assumed to be attributed to inefficiency in the SFA model. To ensure prices were sufficient to finance investment and allow for estimation uncertainty, some premium may need to be added, but there is no well defined methodology for determining such a premium.

Table 5.4: **Model 7: Predicted prices by market segment ($/month)**

Capacity class	*Route type*	*Actual*	*DAA 2012*	*Model (7a)*	*Model (7b)*	*Models (7c)*
Regulated routes						
Capacity < 5 Mbps	Intercapital	1209	1365	421	460	495
	Metro	378	458	216	240	261
	Regional	1043	782	275	308	337
	All (excl. TEs)	713	635	249	278	303
Capacity 5 - 50 Mbps	Intercapital	2881	6192	929	973	989
	Metro	1074	1676	441	469	514
	Regional	3995	3409	663	719	798
	All (excl. TEs)	2385	2807	577	617	672
Capacity 50 - 200 Mbps	Intercapital	6677	13969	1373	1451	1370
	Metro	1524	6405	895	942	1019
	Regional	6129	11946	1211	1287	1363
	All (excl. TEs)	3818	9253	1060	1122	1188
Capacity 200+ Mbps	Intercapital	11784	52403	2199	2303	2116
	Metro	2023	13770	1229	1291	1380
	Regional	5198	36426	1723	1833	1869
	All (excl. TEs)	3834	24883	1479	1562	1614
All capacities	Intercapital	4437	12861	1021	1081	1051
	Metro	824	3106	467	500	540
	Regional	2472	6116	590	639	680
	All (excl. TEs)	1718	4877	546	587	625
	Tail-end	[redacted]	1101	262	290	315
	All (incl. TEs)		3627	452	489	523
Deregulated						
Capacity < 5 Mbps	Intercapital	812	1340	396	431	458
	Metro	398	440	215	239	258
	Regional	504	653	250	280	295
	All (excl. TEs)	434	516	230	255	274
Capacity 5 - 50 Mbps	Intercapital	1534	4853	792	833	845
	Metro	859	1489	437	467	500
	Regional	1884	4294	705	751	832
	All (excl. TEs)	1010	2161	507	539	570
Capacity 50 - 200 Mbps	Intercapital	2432	13215	1364	1431	1392
	Metro	1399	5202	840	886	947
	Regional	3475	10279	1119	1189	1252
	All (excl. TEs)	1714	6723	935	986	1033
Capacity 200+ Mbps	Intercapital	7621	120388	2778	2925	2606
	Metro	1979	14698	1261	1328	1403

Capacity class	*Route type*	*Actual*	*DAA 2012*	*Model (7a)*	*Model (7b)*	*Models (7c)*
	Regional	4449	32605	1674	1787	1809
	All (excl. TEs)	3405	38549	1619	1708	1694
All capacities	Intercapital	3290	38367	1394	1469	1380
	Metro	923	3673	538	573	612
	Regional	2009	9274	748	804	836
	All (excl. TEs)	1307	8433	661	704	726
	Tail-end	.	.	.	.	.
	All (incl. TEs)	1307	8433	661	704	726
Regulated & Deregulated						
Capacity < 5 Mbps	Intercapital	979	1350	406	443	473
	Metro	388	449	216	240	260
	Regional	980	767	272	305	332
	All (excl. TEs)	607	590	242	269	292
Capacity 5 - 50 Mbps	Intercapital	1842	5160	824	865	878
	Metro	911	1535	438	468	503
	Regional	3667	3546	669	724	803
	All (excl. TEs)	1465	2374	530	565	604
Capacity 50 - 200 Mbps	Intercapital	3689	13438	1367	1437	1386
	Metro	1445	5642	860	906	973
	Regional	5645	11642	1194	1269	1343
	All (excl. TEs)	2700	7909	994	1049	1106
Capacity 200+ Mbps	Intercapital	8311	109128	2682	2822	2525
	Metro	1995	14363	1250	1315	1394
	Regional	4998	35407	1710	1820	1853
	All (excl. TEs)	3583	32878	1561	1647	1661
All capacities	Intercapital	3610	31261	1291	1361	1289
	Metro	882	3439	509	543	582
	Regional	2404	6581	613	663	703
	All (excl. TEs)	1521	6579	601	643	673
	Tail-end	[redacted]	1101	262	290	315
	All (incl. TEs)		5455	532	571	600

Table 5.5: **Model 7: Difference in predicted prices compared to DAA 2012 (%)**

Capacity class	*Route type*	*Model (7a)*	*Model (7b)*	*Models (7c)*
Regulated routes				
Capacity < 5 Mbps	Intercapital	-69	-66	-64
	Metro	-53	-48	-43
	Regional	-65	-61	-57
	All (excl. TEs)	-61	-56	-52
Capacity 5 - 50 Mbps	Intercapital	-85	-84	-84
	Metro	-74	-72	-69
	Regional	-81	-79	-77
	All (excl. TEs)	-79	-78	-76
Capacity 50 - 200 Mbps	Intercapital	-90	-90	-90
	Metro	-86	-85	-84
	Regional	-90	-89	-89
	All (excl. TEs)	-89	-88	-87
Capacity 200+ Mbps	Intercapital	-96	-96	-96
	Metro	-91	-91	-90
	Regional	-95	-95	-95
	All (excl. TEs)	-94	-94	-94
All capacities	Intercapital	-92	-92	-92
	Metro	-85	-84	-83
	Regional	-90	-90	-89
	All (excl. TEs)	-89	-88	-87
	Tail-end	-76	-74	-71
	All (incl. TEs)	-88	-87	-86
Deregulated				
Capacity < 5 Mbps	Intercapital	-70	-68	-66
	Metro	-51	-46	-41
	Regional	-62	-57	-55
	All (excl. TEs)	-55	-51	-47
Capacity 5 - 50 Mbps	Intercapital	-84	-83	-83
	Metro	-71	-69	-66
	Regional	-84	-83	-81
	All (excl. TEs)	-77	-75	-74
Capacity 50 - 200 Mbps	Intercapital	-90	-89	-89
	Metro	-84	-83	-82
	Regional	-89	-88	-88
	All (excl. TEs)	-86	-85	-85
Capacity 200+ Mbps	Intercapital	-98	-98	-98
	Metro	-91	-91	-90
	Regional	-95	-95	-94
	All (excl. TEs)	-96	-96	-96
All capacities	Intercapital	-96	-96	-96
	Metro	-85	-84	-83
	Regional	-92	-91	-91
	All (excl. TEs)	-92	-92	-91
	Tail-end	.	.	.
	All (incl. TEs)	-92	-92	-91

Capacity class	*Route type*	*Model (7a)*	*Model (7b)*	*Models (7c)*
Regulated & Deregulated				
Capacity < 5 Mbps	Intercapital	-70	-67	-65
	Metro	-52	-47	-42
	Regional	-64	-60	-57
	All (excl. TEs)	-59	-54	-50
Capacity 5 - 50 Mbps	Intercapital	-84	-83	-83
	Metro	-71	-70	-67
	Regional	-81	-80	-77
	All (excl. TEs)	-78	-76	-75
Capacity 50 - 200 Mbps	Intercapital	-90	-89	-90
	Metro	-85	-84	-83
	Regional	-90	-89	-88
	All (excl. TEs)	-87	-87	-86
Capacity 200+ Mbps	Intercapital	-98	-97	-98
	Metro	-91	-91	-90
	Regional	-95	-95	-95
	All (excl. TEs)	-95	-95	-95
All capacities	Intercapital	-96	-96	-96
	Metro	-85	-84	-83
	Regional	-91	-90	-89
	All (excl. TEs)	-91	-90	-90
	Tail-end	-76	-74	-71
	All (incl. TEs)	-90	-90	-89

Table 5.6: **Model 7: Difference in predicted prices compared to Actual (%)**

Capacity class	*Route type*	*DAA 2012*	*Model (7a)*	*Model (7b)*	*Models (7c)*
Regulated routes					
Capacity < 5 Mbps	Intercapital	13	-65	-62	-59
	Metro	21	-43	-36	-31
	Regional	-25	-74	-70	-68
	All (excl. TEs)	-11	-65	-61	-57
Capacity 5 - 50 Mbps	Intercapital	115	-68	-66	-66
	Metro	56	-59	-56	-52
	Regional	-15	-83	-82	-80
	All (excl. TEs)	18	-76	-74	-72
Capacity 50 - 200 Mbps	Intercapital	109	-79	-78	-79
	Metro	320	-41	-38	-33
	Regional	95	-80	-79	-78
	All (excl. TEs)	142	-72	-71	-69
Capacity 200+ Mbps	Intercapital	345	-81	-80	-82
	Metro	581	-39	-36	-32
	Regional	601	-67	-65	-64
	All (excl. TEs)	549	-61	-59	-58
All capacities	Intercapital	190	-77	-76	-76
	Metro	277	-43	-39	-34
	Regional	147	-76	-74	-72
	All (excl. TEs)	184	-68	-66	-64

Capacity class	*Route type*	*DAA 2012*	*Model (7a)*	*Model (7b)*	*Models (7c)*
	Tail-end	166	-37	-30	-24
	All (incl. TEs)	182	-65	-62	-59
Deregulated					
Capacity < 5 Mbps	Intercapital	65	-51	-47	-44
	Metro	10	-46	-40	-35
	Regional	29	-50	-44	-41
	All (excl. TEs)	19	-47	-41	-37
Capacity 5 - 50 Mbps	Intercapital	216	-48	-46	-45
	Metro	73	-49	-46	-42
	Regional	128	-63	-60	-56
	All (excl. TEs)	114	-50	-47	-44
Capacity 50 - 200 Mbps	Intercapital	443	-44	-41	-43
	Metro	272	-40	-37	-32
	Regional	196	-68	-66	-64
	All (excl. TEs)	292	-45	-42	-40
Capacity 200+ Mbps	Intercapital	1480	-64	-62	-66
	Metro	643	-36	-33	-29
	Regional	633	-62	-60	-59
	All (excl. TEs)	1032	-52	-50	-50
All capacities	Intercapital	1066	-58	-55	-58
	Metro	298	-42	-38	-34
	Regional	362	-63	-60	-58
	All (excl. TEs)	545	-49	-46	-44
	Tail-end	.	.	.	.
	All (incl. TEs)	545	-49	-46	-44
Regulated & Deregulated					
Capacity < 5 Mbps	Intercapital	38	-58	-55	-52
	Metro	16	-44	-38	-33
	Regional	-22	-72	-69	-66
	All (excl. TEs)	-3	-60	-56	-52
Capacity 5 - 50 Mbps	Intercapital	180	-55	-53	-52
	Metro	68	-52	-49	-45
	Regional	-3	-82	-80	-78
	All (excl. TEs)	62	-64	-61	-59
Capacity 50 - 200 Mbps	Intercapital	264	-63	-61	-62
	Metro	291	-40	-37	-33
	Regional	106	-79	-78	-76
	All (excl. TEs)	193	-63	-61	-59
Capacity 200+ Mbps	Intercapital	1213	-68	-66	-70
	Metro	620	-37	-34	-30
	Regional	608	-66	-64	-63
	All (excl. TEs)	818	-56	-54	-54
All capacities	Intercapital	766	-64	-62	-64
	Metro	290	-42	-38	-34
	Regional	174	-74	-72	-71
	All (excl. TEs)	332	-60	-58	-56
	Tail-end	166	-37	-30	-24
	All (incl. TEs)	322	-59	-56	-54

6 DISCUSSION

This section summarises the comparative performance of the models. The goodness-of-fit statistics of the models are compared (Table 6.1), and the predictive results of the model are discussed, focussing first on the predictions of the new models for the deregulated routes (Table 6.2), then the predictions for regulated routes based on the DAA 2012 model (Table 6.3) and then the predictions of actual prices for the regulated routes (Table 6.4).

As shown in our earlier report (Economic Insights 2015), and in this report, various versions of the new models are considered to be an improvement on the DAA 2012 model. So the focus is on the relative performance of the new models and the various adjustments that were made to address issues raised about their capacity to forecast actual prices particularly for low capacity short distance routes.

Section 2.3 of this report explained that in assessing the predictive capacity of the models most of the emphasis should be on how well the models forecast prices for deregulated routes. This follows since the models are based on the deregulated market and then used to obtain a benchmark 'competitive' price for the regulated market. If the new models provide superior predictions relative to the DAA 2012 model for the deregulated market they should be preferred to the DAA 2012 model for application to regulated routes. As noted, if the new models provide lower predictions for the regulated market than the DAA 2012 model but these predictions are not good predictors of the actual prices in the regulated market that is of limited consequence because commercial negotiation can lead to actual prices in the regulated market that are less than the regulated prices.

In considering the predictive capacity, statistical measures of goodness of fit and prediction errors are relevant but need to be supported by also examining the predictions for different routes. Another consideration in assessing the models is whether the estimated cost elasticities accord with economic theory which predicts that increases in speed or distance should have a positive effect (because they are regarded as outputs) and that if there are scale effects the elasticities should be between 0 and 1.

Table 6.1 summarises the key goodness-of-fit statistics for models 1 to 7. Table 6.2 presents a comparison of key predictions of the new models for deregulated routes relative to actual prices. It also includes comments on the signs of cost elasticities. Table 6.3 presents a comparison of the key predictions for regulated routes relative to current regulated prices (i.e. the predictions of the DAA 2012 model) while Table 6.4 shows the comparison against actual prices.

Model 1 is the basis against which the other models are compared. The out-of-sample goodness-of-fit is a more reliable basis for comparison where available. In models (1a) and (1b) the out-of-sample goodness-of-fit is similar to the full sample goodness-of-fit, but this is not always the case for the other models.

Inspection of Table 6.1 highlights the following:

- All of the models exhibit broadly similar goodness of fit and differences are small and not a basis by themselves to clearly prefer one model over another.

- For models (2b) and (2c) where the extremely influential observations have been excluded, there is an improvement in the within-sample goodness-of-fit measures, but this provides a misleading guide to these models' predictive ability. The out-of-sample goodness-of-fit statistics show deterioration in the predictive performance relative to Model 1, when the extremely influential observations are removed.

Table 6.1: **Comparison of goodness-of-fit measures**

	Out-of-sample*			**Full sample**		
Model	*R-sq*	*RMSE*	*MAE*	*R-sq*	*RMSE*	*MAE*
Model type (a)						
Model 1a	0.68332	0.50702	0.36410	0.68472	0.50597	0.36364
Model 2a	0.67839	0.51145	0.36569	0.71762	0.45393	0.33467
Model 3a	na	na	na	0.68015	0.50939	0.35614
Model 4a	0.69411	0.49818	0.35894	0.69585	0.49674	0.35796
Model 5a	0.68813	0.50311	0.35689	0.68965	0.50197	0.35620
Model 6a	0.68535	0.50534	0.36012	0.68707	0.50407	0.35939
Model 7a	na	na	na	0.68380	na	Na
Model type (b)						
Model 1b	0.68014	0.50947	0.36474	0.68149	0.50850	0.36430
Model 2b	0.67554	0.51372	0.36673	0.71562	0.45552	0.33541
Model 3b	na	na	na	0.67740	0.51155	0.35640
Model 4b	0.68703	0.50373	0.36156	0.68877	0.50237	0.36072
Model 5b	0.68507	0.50545	0.35823	0.68654	0.50439	0.35764
Model 6b	0.68168	0.50815	0.36166	0.68338	0.50696	0.36104
Model 7b	na	na	na	0.68024	na	na
Model type (c)						
Model 1c	0.67294	0.51584	0.37395	0.67423	0.51504	0.37337
Model 2c	0.66886	0.51981	0.37486	0.70913	0.46155	0.34352
Model 3c	na	na	na	0.66792	0.51970	0.36958
Model 4c	0.68253	0.50852	0.36624	0.68410	0.50746	0.36550
Model 5c	0.67879	0.51110	0.36634	0.68015	0.51024	0.36571
Model 6c	0.67473	0.51434	0.37051	0.67636	0.51334	0.36975
Model 7c	na	na	na	0.67721	na	na

Notes: * Out-of-sample statistics are based on smaller samples than the full sample statistics.

RMSE refers to the "root mean squared error" and MAE refers to the "mean absolute error". Denoting the difference between the predicted and actual values for observation *i* (i.e. the residual) as: r_i then RMSE = $\sqrt{\sum_i r_i^2}$, and MAE = $(1/n)\sum_i |r_i|$, where *n* is the number of observations. The RMSE gives a relatively higher weight to large errors.

- Model 3 uses the Robust RE method of estimation, and these models were only fitted using the full sample so only within-sample goodness-of-fit measures are available. For these measures Model 3's goodness-of-fit was similar to those of Model 1. More specifically, the R^2 is lower and the RMSE is higher, but its MAE is lower. These

results suggest that the robust regression random effects method (model 3) does not represent a clear improvement over model 1, which uses the standard random effects method.

- Model 4 was estimated by excluding services < 2.5Mbps and < 5km from the sample for estimating the regression. An alternative method was used for predicting the prices of those services, based on an average cost of all services < 2.5Mbps and < 5km on deregulated routes. The goodness-of-fit measures for model 4 shown here are calculated for the whole data sample using the predicted values from both these sources. Models 4b and 4c are both an improvement in fit compared to the corresponding versions of model 1. The method used in Model 4 is, on the basis of goodness-of-fit, a feasible alternative to Model 1.
- Model 5 includes a specific effect in the regression for services of < 2.5 Mbps and < 5km. The out-of-sample goodness-of-fit is similar to that of model 4, and represents a slight improvement over model 1.
- Model 6 is the piecewise regression that allows for differential coefficients on the two output variables for services < 2.5 Mbps and < 5 km, compared to services of higher capacity or length. The out-of-sample goodness-of-fit of Model 6 is similar to Model 1, with a very slight improvement. The out-of-sample goodness-of-fit is slightly less than for models 4 and 5.
- Most of the measures of goodness-of-fit are not available for the stochastic frontier method (model 7). This is because for all other models the RMSE and MAE are calculated using the combined residual which includes both the random effects and the ordinary errors. In the stochastic frontier model, instead of random effects being normally distributed with a zero mean, the corresponding stochastic component is strictly positive and with a half-normal distribution. Consequently, when combined with the ordinary errors, the results will be a highly skewed distribution of residuals. The RMSE's and MAE's calculated from them would not be comparable with those reported for the other models. The full sample R^2's for the stochastic frontier models are very similar to those for Model 1. These results suggest that the stochastic frontier method does not provide a clear improvement over model 1. Further, the stochastic frontier models have other shortcomings which have been noted above. The concern is that it is not clear the methodology is appropriate if the deregulated market is considered to be competitive on average and an average competitive price is considered to be the most suitable benchmark.

Table 6.2 presents a comparison of the key predictions for deregulated routes relative to actual prices. It is important to note that the models used for prediction are those simplified according to the methods described in our final report (Economic Insights 2015). Inspection of the table highlights the following:

- Across all routes (i.e. on average over all services) almost all of the models predict prices that are less than average actual prices for deregulated routes. The two exceptions are models 4b and 4c, where the average predicted prices on all deregulated routes are approximately equal to the actual average prices.

- Generally versions (b) and (c) of the models produce more accurate predictions for deregulated low capacity metro routes than version (a).
- It is also the case that for deregulated intercapital and regional routes all of the models predicted prices that are on average substantially lower than the actual prices.
- Model 7 provides forecasts for average prices over all deregulated routes that are substantially less than actual prices — between 44 and 49 per cent lower.

Table 6.3 presents a comparison of the key predictions for regulated routes relative to current regulated prices resulting from application of the DAA 2012 model. Inspection of the table highlights that:

- In all of the market segments shown, the average predicted prices are substantially lower than the current regulated prices.
- Most of the models produce similar percentage differences between average predicted prices and current regulated prices. The exception is that Model 7 (the stochastic frontier model) results in very large differences between average predicted prices and current regulated prices. Over all regulated services, the stochastic frontier models produce 86-88 per cent lower predicted prices than DAA 2012 model on which current regulated prices are based.

Table 6.4 presents a comparison of the key predictions for regulated routes relative to actual prices. These comparisons are of lesser importance because where actual prices on regulated routes differ from regulated prices, this suggests that parties have opted to agree to different prices, and any changes in regulated prices would not directly alter such agreements. Inspection of Table 6.4 highlights the following:

- Across all routes **on average** all models predicted prices that are substantially less than actual prices for regulated routes.
- This is also the case for regulated intercapital and regional routes on average.
- All models except the stochastic frontier model (model 7) predict higher prices than actual prices for the regulated tail-end routes.
- All of the (b) and (c) versions of the models except for model 7 predict prices that are higher than actual prices on regulated low capacity (and in some cases high capacity) metro routes.
- Model 5 which includes a specific effect to represent a step change for capacity < 2.5 Mbps and ≥ 2.5 Mbps provides forecasts for low capacity metro services that are 7 per cent less than actual prices for model 5(a) and 12 per cent higher for model 5(c) and forecasts for high capacity metro services that range from 9 per cent less to 3 per cent higher than actual prices for regulated routes for models 5(a) and 5(c) respectively.
- Model 7 provides forecasts that are substantially less than actual prices for all regulated routes.

As explained in Section 2.3 we consider that most weight should be given to the predictive performance for deregulated routes, including both overall predictive capacity, as well as

minimising differences for different route categories. We also think that the sign and size of the cost elasticities also need to accord with economic theory.

This report has tested various alternatives to the base case specifications represented by Model 1. Generally, the base case models have performed well against these alternatives. Among the alternatives, models 4c and 5c have provided the best predictive performance, while at the same time being reasonably "well behaved" in terms of minimising differences for different route categories. Although Model 4c is very marginally better than 5c on the basis of out of sample MAE and RMSE, there appears to be a problem associated with negative elasticities in some relevant output ranges.

Overall we consider that 5c is simpler than 4c, and it avoids discontinuities that may arise from using different price formulas for low capacity short distance services and other services. Given these considerations we consider that models 1b, 1c or 5c are most suitable for predicting benchmark 'competitive' prices for regulated routes.

Table 6.2 **Comparison of key predictions of the new models relative to actual prices: deregulated routes**

Model	*<5Mbps metro*	*>200mbps metro*	*Intercapital & Regional*	*All*	*Comment on cost elasticities*
Model 1 (same spec. as Table 5.1 Sep. 2015 report)	15% lower for 1a 3% lower for 1b 2% higher for 1c	3% lower for 1a 4% higher for 1b 8% higher for 1c	Intercapital 19-24% lower on average Regional 21-31% lower on average	18% lower for 1a 11% lower for 1b 9% lower for 1c	Cost elasticities are positive for capacity and distance.
Model 2 excludes outliers	12% lower for 2a 5% lower for 2b 0 % difference for 2c	4% lower for 2a 1% higher for 2b 4% higher for 2c	Intercapital 17-20% lower on average Regional 22-29% lower on average	17% lower for 2a 13% lower for 2b 11% lower for 2c	Cost elasticities are positive for capacity and distance.
Model 3 robust RE regression	12% lower for 3a 3% lower for 3b 2% higher for 3c	5% lower for 3a 1% lower for 3b 4% higher for 3c	Intercapital 15-19% lower on average Regional 27-36% lower on average	19% lower for 3a 14% lower for 3b 11% lower for 3c	Cost elasticities are positive for capacity and distance.
Model 4 excludes < 2.5 Mbps & < 5km (replaced with average price for those services)	12% lower for 4a 1% higher for 4b 5% higher for 4c	3% higher for 4a 17% higher for 4b 18% higher for 4c	Intercapital 9-19% lower on average Regional 17-30% lower on average	13% lower for 4a 0% difference for 4b 0% difference for 4c	Cost elasticities with respect to distance are negative for small capacity/distances for all models on deregulated routes, and for models 4a and 4b on regulated routes. But for small-scale services the econometric model may not be setting the price.

Model	*<5Mbps metro*	*>200mbps metro*	*Intercapital & Regional*	*All*	*Comment on cost elasticities*
Model 5 includes specific effect for services < 2.5 Mbps & < 5 km	14% lower for 5a 2% lower for 5b 3% higher for 5c	5% lower for 5a 2% higher for 5b 6% higher for 5c	Intercapital 16-21% lower on average Regional 22-32% lower on average	17% lower for 5a 10% lower for 5b 8% lower for 5c	Cost elasticities with respect to distance are negative for small capacity/distance services for models 5a and 5b on deregulated routes only. Cost elasticities are positive for capacity and distance for 5c on both regulated and deregulated routes.
Model 6 piecewise regression	15% lower for 6a 3% lower for 6b 2% higher for 6c	4% lower for 6a 2% higher for 6b 6% higher for 6c	Intercapital 17-21% lower on average Regional 21-31% lower on average	17% lower for 6a 10% lower for 6b 8% lower for 6c	Cost elasticities with respect to capacity are negative for low capacities for models 6b and 6c but not 6a.
Model 7 stochastic frontier	46% lower for 7a 40% lower for 7b 35% lower for 7c	36% lower for 7a 33% lower for 7b 29% lower for 7c	Intercapital 55-58% lower on average Regional 58-63% lower on average	49% lower for 7a 46% lower for 7b 44% lower for 7c	Cost elasticities are positive for capacity and distance.

Table 6.3 **Comparison of key predictions of the new models relative to DAA 2012 prices: regulated routes**

Model	*<5Mbps metro*	*>200mbps metro*	*Tail end*	*Intercapital & Regional*	*All excluding tail ends*	*All including tail ends*
Model 1 (same spec. as Table 5.1 Sep. 2015 report)	24% lower for 1a 14% lower for 1b 9% lower for 1c	86% lower for 1a 85% lower for 1b 84% lower for 1c	61% lower for 1a 56% lower for 1b 54% lower for 1c	Intercapital 85-86% lower on average Regional 79-82% lower on average	81% lower for 1a 79% lower for 1b 78% lower for 1c	79% lower for 1a 77% lower for 1b 75% lower for 1c
Model 2 excludes outliers	22% lower for 2a 15% lower for 2b 10% lower for 2c	86% lower for 2a 85% lower for 2b 85% lower for 2c	61% lower for 2a 58% lower for 2b 56% lower for 2c	Intercapital 85-86% lower on average Regional 79-81% lower on average	80% lower for 2a 79% lower for 2b 78% lower for 2c	78% lower for 2a 77% lower for 2b 76% lower for 2c
Model 3 robust regression	22% lower for 3a 14% lower for 3b 9% lower for 3c	87% lower for 3a 86% lower for 3b 85% lower for 3c	61% lower for 3a 58% lower for 3b 56% lower for 3c	Intercapital 85–86% lower on average Regional 80-83% lower on average	82% lower for 2a 80% lower for 2b 79% lower for 2c	80% lower for 2a 78% lower for 2b 77% lower for 2c
Model 4 excludes < 2.5 Mbps & < 5km	23% lower for 4a 9% lower for 4b 6% lower for 4c	86% lower for 4a 84% lower for 4b 84% lower for 4c	53% lower for 4a 43% lower for 4b 40% lower for 4c	Intercapital 83-86% lower on average Regional 78-82% lower on average	80% lower for 4a 77% lower for 4b 76% lower for 4c	78% lower for 4a 74% lower for 4b 73% lower for 4c
Model 5 includes specific effect for 2.5 Mbps	24% lower for 5a 13% lower for 5b 8% lower for 5c	87% lower for 5a 86% lower for 5b 85% lower for 5c	56% lower for 5a 50% lower for 5b 48% lower for 5c	Intercapital 85-86% lower on average Regional 79-82% lower on average	81% lower for 5a 79% lower for 5b 78% lower for 5c	78% lower for 5a 76% lower for 5b 75% lower for 5c

Model	*<5Mbps metro*	*>200mbps metro*	*Tail end*	*Intercapital & Regional*	*All excluding tail ends*	*All including tail ends*
Model 6 piecewise regression	25% lower for 6a	86% lower for 6a	62% lower for 6a	Intercapital 85-86% lower on average	81% lower for 6a	79% lower for 6a
	14% lower for 6b	85% lower for 6b	57% lower for 6b		79% lower for 6b	77% lower for 6b
	9% lower for 6c	85% lower for 6c	55% lower for 6c	Regional 79-82% lower on average	78% lower for 6c	75% lower for 6c
Model 7 stochastic frontier	53% lower for 7a	91% lower for 7a	76% lower for 7a	Intercapital 92% lower on average	89% lower for 7a	88% lower for 7a
	48% lower for 7b	91% lower for 7b	74% lower for 7b		88% lower for 7b	87% lower for 7b
	43% lower for 7c	90% lower for 7c	71% lower for 7c	Regional 89-90% lower on average	87% lower for 7c	86% lower for 7c

Table 6.4 **Comparison of key predictions of the new models relative to actual prices: regulated routes**

Model	*<5Mbps metro*	*>200mbps metro*	*Tail end*	*Intercapital & Regional*	*All excluding tail ends*	*All including tail ends*
Model 1 (same spec. as Table 5.1 Sep. 2015 report)	8% lower for 1a 5% higher for 1b 10% higher for 1c	5% lower for 1a 1% higher for 1b 7% higher for 1c	4% higher for 1a 17% higher for 1b 22% higher for 1c	Intercapital 57-60% lower on average Regional 48-56% lower on average	45% lower for 1a 40% lower for 1b 37% lower for 1c	40% lower for 1a 34% lower for 1b 30% lower for 1c
Model 2 excludes outliers	5% lower for 2a 3% higher for 2b 9% higher for 2c	5% lower for 2a 1% lower for 2b 4% higher for 2c	4% higher for 2a 12% higher for 2b 17% higher for 2c	Intercapital 57-59% lower on average Regional 48-54% lower on average	44% lower for 2a 41% lower for 2b 38% lower for 2c	39% lower for 2a 35% lower for 2b 32% lower for 2c
Model 3 robust regression	5% lower for 3a 4% higher for 3b 10% higher for 3c	9% lower for 3a 5% lower for 3b 2% higher for 3c	3% higher for 3a 12% higher for 3b 18% higher for 3c	Intercapital 57–60% lower on average Regional 51-59% lower on average	48% lower for 2a 44% lower for 2b 40% lower for 2c	42% lower for 2a 38% lower for 2b 34% lower for 2c
Model 4 excludes < 2.5 Mbps & < 5km	7% lower for 4a 10% higher for 4b 14% higher for 4c	4% lower for 4a 9% higher for 4b 12% higher for 4c	25% higher for 4a 51% higher for 4b 59% higher for 4c	Intercapital 52-59% lower on average Regional 45-55% lower on average	44% lower for 4a 35% lower for 4b 33% lower7 for 4c	37% lower for 4a 26% lower for 4b 23% lower for 4c
Model 5 includes specific effect for 2.5 Mbps	7% lower for 5a 6% higher for 5b 12% higher for 5c	9% lower for 5a 2% lower for 5b 3% higher for 5c	18% higher for 5a 32% higher for 5b 39% higher for 5c	Intercapital 56-60% lower on average Regional 48-55% lower on average	46% lower for 5a 40% lower for 5b 37% lower for 5c	39% lower for 5a 32% lower for 5b 29% lower for 5c

Model	*<5Mbps metro*	*>200mbps metro*	*Tail end*	*Intercapital & Regional*	*All excluding tail ends*	*All including tail ends*
Model 6 piecewise regression	9% lower for 6a 5% higher for 6b 10% higher for 6c	7% lower for 6a 0 % difference between predicted and actual for 6b 5% higher for 6c	2% higher for 6a 15% higher for 6b 21% higher for 6c	Intercapital 57-60% lower on average Regional 47-55% lower on average	45% lower for 6a 40% lower for 6b 36% lower for 6c	40% lower for 6a 34% lower for 6b 30% lower for 6c
Model 7 stochastic frontier	43% lower for 7a 36% lower for 7b 31% lower for 7c	39% lower for 7a 36% lower for 7b 32% lower for 7c	37% lower for 7a 30% lower for 7b 24% lower for 7c	Intercapital 76-77% lower on average Regional 72-76% lower on average	68% lower for 7a 66% lower for 7b 64% lower for 7c	65% lower for 7a 62% lower for 7b 59% lower for 7c

REFERENCES

CEG 2015, 'Review of the draft decision on DTCS FAD', in *Competition Economics Group report prepared for Optus*.

CRAN 2015, 'Package 'robustlmm''.

Data Analysis Australia (DAA) 2012, 'Domestic Transmission Capacity Service Price Benchmarking - Pricing Model Development: Consolidated Report', prepared for the Australian Competition and Consumer Commission.

Economic Insights 2015, 'Domestic Transmission Capacity Services Benchmarking Model: Final Report prepared for Australian Competition and Consumer Commission'.

Greene, W. H. 2012, *Econometric Analysis*, Seventh (International) edn, Pearson.

Koller, M. 2013, Robust Estimation of Linear Mixed Models, ETH Zurich.

Koller, M. 2015, 'robustlmm: Robust Estimating Equations and Examples'.

Verardi, V. & Croux, C. 2009, 'Robust regression in Stata', *Stata Journal*, vol. 9, no. 3, pp. 439-53.

APPENDIX A: RESULTS EXCLUDING ADDITIONAL DATA

This appendix presents the results of re-estimating Models 1 to 6 using a smaller data sample which excludes the additional data that was provided relatively late in the process.

A1 Model 1: Base models

Table A.1: **Model A1: Base case RE models (2014 data excl. new obs.)**

	Model (A1a)		**Model (A1b)**		**Model (A1c)**	
Predictor	*Coeff.*	*t-stat.*	*Coeff.*	*t-stat.*	*Coeff.*	*t-stat.*
constant	5.54896	33.18	4.65796	40.26	4.85811	56.46
log capacity	0.49799	43.07	0.49239	42.52	0.49147	42.51
log distance	0.10059	4.25	0.09745	4.09	0.11703	4.97
0.5(log capacity)^2	-0.03599	-12.49	-0.03527	-12.20	-0.03503	-12.13
0.5(log distance)^2	0.01269	1.95	0.01319	2.02	0.01295	1.97
(log capacity)(log distance)	-0.00375	-2.47	-0.00367	-2.41	-0.00471	-3.17
log route t'put	-0.01810	-3.31	-0.01965	-3.58	.	.
log ESA t'put	0.03576	3.82	0.03215	3.42	.	.
contract start date	-0.00005	-7.34	.	.	.	.
route class 2 (Metro)	0.17378	2.35	0.17682	2.38	0.22021	2.98
route class 3 (Regional)	0.31945	5.48	0.31568	5.39	0.32620	5.54
Provider #1						
Provider #3						
Provider #4						
Provider #5						
Provider #6						
Provider #7						
Provider #8						
Provider #9						
interface-type 3 (SDH)	0.20452	10.55	0.24336	13.00	0.24014	12.83
σ(u)	0.31729		0.31938		0.32467	
σ(e)	0.42752		0.42906		0.42864	

Source: Economic Insights estimation results.

Notes: * Squared correlation between fitted and actual dependent.

Table A.2: **Model A1: Base case models (2014 data excl. new obs.): Goodness-of-fit**

Goodness-of-fit statistic	**Model (A1a)** *10-fold validation (avg)*	*Full sample*	**Model (A1b)** *10-fold validation (avg)*	*Full sample*	**Model (A1c)** *10-fold validation (avg)*	*Full sample*
Within-sample						
obs	6090	6767	6090	6767	6090	6767
R^2*	0.69330	0.69272	0.69157	0.69098	0.68691	0.68644
BIC		9312.5		9357.3		9355.6
RMSE (based on ue)	0.52521	0.52579	0.52661	0.52718	0.53110	0.53162
MAE (based on ue)	0.37129	0.37168	0.37168	0.37208	0.37655	0.37673
Out-of-sample						
obs	677	0	677	0	677	0
R^2*	0.69159		0.68987		0.68543	
RMSE (based on ue)	0.52674		0.52809		0.53236	
MAE (based on ue)	0.37248		0.37277		0.37760	

Notes: * Squared correlation between fitted and actual dependent.

Table A.3: **Model A1: Base case models (2014 data excl. new obs.): Statistical tests**

	Model (A1a) *Stat.*	*P-value**	**Model (A1b)** *Stat.*	*P-value**	**Model (A1c)** *Stat.*	*P-value**
Normality of residuals						
Doornik-Hansen[(1)]	4013.9	0.0000	4138.1	0.0000	4084.0	0.0000
IQR (% severe outliers) [(2)†]	1.79%		1.91%		1.81%	
Homoscedasticity						
Breusch-Pagan/Cook-Weisberg[(6)]	1325.4	0.0000	1380.9	0.0000	1444.6	0.0000
Multicollinearity						
# VIF scores > 10	5/19		5/18		5/16	
Misspecification						
RESET [(7)]	16.1	0.0000	11.0	0.0000	12.0	0.0000
Link test[(8)]	2.20	0.028	0.91	0.362	0.77	0.439
Joint significance tests						
2nd order output terms (df = 3)	198.5	0.0000	189.8	0.0000	196.9	0.0000
Route classes (df = 2)	41.7	0.0000	39.5	0.0000	36.9	0.0000
Provider fixed effects (df = 7)	1090.3	0.0000	1107.8	0.0000	1109.3	0.0000

Note: * Null hypothesis is rejected, as a standard procedure, in these tests, if P-value is less than 0.05. Equivalently, the reported statistic exceeds the critical value for that statistic; † Percentage of n = 6767 observations; (1) $chi^2(2k)$ where k = 19 for 1st model, and k = 18 for 2nd model and k = 16 for 3rd model. (2) Severe outliers represent about 0.0002% of a normal distribution; (3) Studentized residual > 3; (4) Hat value > $3k/n$; (5) Cook's D > 5 × average Cook's D; (6) $chi^2(1)$; (7) Via powers of the dependent variable, $F(3,n–k–3)$; (8) t-statistic on hat^2; (9) $F(r,n–k–r)$, where r = number of parameters tested, and r = 3 for higher-order output terms, r = 2 for route classes, and r = 8 for provider-specific effects. (10) $chibar^2(1)$; a Approximate, based on OLS regression of $(y - \hat{u}_i)$ on the predictors.

Table A.4: **Model A1: Average predicted prices by market segment ($/month)**

Capacity class	*Route type*	*Actual*	*DAA 2012*	*DAA 2015*	*Model (A1a)*	*Model (A1b)*	*Models (A1c)*
Regulated routes							
Capacity < 5 Mbps	Intercapital	1209	1365	949	723	791	808
	Metro	375	457	427	366	408	423
	Regional	1043	781	598	563	625	654
	All (excl. TEs)	713	635	522	468	520	542
Capacity 5 - 50 Mbps	Intercapital	2881	6192	2093	1618	1708	1715
	Metro	1075	1678	906	770	826	863
	Regional	3995	3409	1350	1397	1514	1598
	All (excl. TEs)	2387	2809	1199	1099	1182	1234
Capacity 50 - 200 Mbps	Intercapital	9260	15157	3310	2754	2949	2904
	Metro	2197	6295	1805	1673	1793	1865
	Regional	9691	13374	2823	2932	3142	3281
	All (excl. TEs)	6388	10441	2421	2368	2537	2632
Capacity 200+ Mbps	Intercapital	14561	61066	6783	4591	4863	4747
	Metro	2684	16947	3034	2574	2746	2850
	Regional	7458	48954	5387	4679	5018	5177
	All (excl. TEs)	5611	33806	4312	3614	3864	3974
All capacities	Intercapital	4842	12774	2420	1820	1943	1929
	Metro	782	2321	793	687	747	776
	Regional	2630	5544	1216	1158	1259	1313
	All (excl. TEs)	1811	4267	1060	954	1035	1074
	Tail-end	[redacted]	1064	601	447	492	505
	All (incl. TEs)	[redacted]	3115	895	772	840	869
Deregulated							
Capacity < 5 Mbps	Intercapital	812	1340	941	687	747	761
	Metro	397	439	426	356	396	410
	Regional	504	653	554	497	552	569
	All (excl. TEs)	434	516	470	391	434	447
Capacity 5 - 50 Mbps	Intercapital	1534	4853	1804	1386	1466	1471
	Metro	859	1490	815	709	762	784
	Regional	1901	4346	1507	1489	1589	1677
	All (excl. TEs)	1010	2162	1007	852	911	932
Capacity 50 - 200 Mbps	Intercapital	2474	13244	3089	2467	2610	2590
	Metro	1680	4685	1530	1404	1503	1544
	Regional	4432	10605	2519	2529	2703	2811
	All (excl. TEs)	2090	6940	1932	1720	1834	1869
Capacity 200+ Mbps	Intercapital	7744	122355	9804	5642	5999	5815
	Metro	2514	18773	3076	2532	2703	2773
	Regional	5421	37840	4663	4158	4465	4584
	All (excl. TEs)	4418	52098	5280	3652	3895	3894
All capacities	Intercapital	3316	38645	4130	2660	2827	2776
	Metro	895	3073	933	800	865	889
	Regional	2116	8966	1666	1547	1669	1724
	All (excl. TEs)	1337	8531	1441	1123	1207	1224
	Tail-end	.	.	.	.	.	.
	All (incl. TEs)	1337	8531	1441	1123	1207	1224

Capacity class	*Route type*	*Actual*	*DAA 2012*	*DAA 2015*	*Model (A1a)*	*Model (A1b)*	*Models (A1c)*
Regulated & Deregulated							
Capacity < 5 Mbps	Intercapital	979	1350	944	702	766	781
	Metro	387	448	427	361	402	416
	Regional	980	766	593	555	617	644
	All (excl. TEs)	606	589	502	439	487	506
Capacity 5 - 50 Mbps	Intercapital	1842	5160	1870	1439	1521	1527
	Metro	911	1535	837	724	778	803
	Regional	3675	3552	1374	1411	1525	1610
	All (excl. TEs)	1465	2376	1071	933	1000	1032
Capacity 50 - 200 Mbps	Intercapital	4032	13683	3140	2533	2687	2662
	Metro	1835	5169	1613	1485	1590	1640
	Regional	8561	12778	2758	2845	3047	3180
	All (excl. TEs)	3857	8379	2133	1986	2123	2183
Capacity 200+ Mbps	Intercapital	8686	113887	9387	5497	5842	5667
	Metro	2573	18139	3061	2546	2718	2800
	Regional	6838	45569	5166	4521	4849	4996
	All (excl. TEs)	4874	45105	4910	3637	3883	3924
All capacities	Intercapital	3711	31948	3688	2443	2599	2557
	Metro	849	2764	875	754	817	843
	Regional	2555	6047	1282	1215	1319	1373
	All (excl. TEs)	1584	6310	1242	1035	1118	1146
	Tail-end	[redacted]	1064	601	447	492	505
	All (incl. TEs)	[redacted]	5124	1097	902	976	1001

Table A.5: **Model A1: Difference in predicted prices compared to DAA 2012 (%)**

Capacity class	*Route type*	*Actual*	*DAA 2015*	*Model (A1a)*	*Model (A1b)*	*Models (A1c)*
Regulated routes						
Capacity < 5 Mbps	Intercapital	-11	-30	-47	-42	-41
	Metro	-18	-7	-20	-11	-8
	Regional	34	-24	-28	-20	-16
	All (excl. TEs)	12	-18	-26	-18	-15
Capacity 5 - 50 Mbps	Intercapital	-53	-66	-74	-72	-72
	Metro	-36	-46	-54	-51	-49
	Regional	17	-60	-59	-56	-53
	All (excl. TEs)	-15	-57	-61	-58	-56
Capacity 50 - 200 Mbps	Intercapital	-39	-78	-82	-81	-81
	Metro	-65	-71	-73	-72	-70
	Regional	-28	-79	-78	-77	-75
	All (excl. TEs)	-39	-77	-77	-76	-75
Capacity 200+ Mbps	Intercapital	-76	-89	-92	-92	-92
	Metro	-84	-82	-85	-84	-83
	Regional	-85	-89	-90	-90	-89
	All (excl. TEs)	-83	-87	-89	-89	-88
All capacities	Intercapital	-62	-81	-86	-85	-85
	Metro	-66	-66	-70	-68	-67
	Regional	-53	-78	-79	-77	-76
	All (excl. TEs)	-58	-75	-78	-76	-75

Capacity class	*Route type*	*Actual*	*DAA 2015*	*Model (A1a)*	*Model (A1b)*	*Models (A1c)*
	Tail-end	-62	-44	-58	-54	-53
	All (incl. TEs)	-58	-71	-75	-73	-72
Deregulated						
Capacity < 5 Mbps	Intercapital	-39	-30	-49	-44	-43
	Metro	-10	-3	-19	-10	-7
	Regional	-23	-15	-24	-15	-13
	All (excl. TEs)	-16	-9	-24	-16	-13
Capacity 5 - 50 Mbps	Intercapital	-68	-63	-71	-70	-70
	Metro	-42	-45	-52	-49	-47
	Regional	-56	-65	-66	-63	-61
	All (excl. TEs)	-53	-53	-61	-58	-57
Capacity 50 - 200 Mbps	Intercapital	-81	-77	-81	-80	-80
	Metro	-64	-67	-70	-68	-67
	Regional	-58	-76	-76	-75	-73
	All (excl. TEs)	-70	-72	-75	-74	-73
Capacity 200+ Mbps	Intercapital	-94	-92	-95	-95	-95
	Metro	-87	-84	-87	-86	-85
	Regional	-86	-88	-89	-88	-88
	All (excl. TEs)	-92	-90	-93	-93	-93
All capacities	Intercapital	-91	-89	-93	-93	-93
	Metro	-71	-70	-74	-72	-71
	Regional	-76	-81	-83	-81	-81
	All (excl. TEs)	-84	-83	-87	-86	-86
	Tail-end	.	.	.	.	.
	All (incl. TEs)	-84	-83	-87	-86	-86
Regulated & Deregulated						
Capacity < 5 Mbps	Intercapital	-28	-30	-48	-43	-42
	Metro	-14	-5	-19	-10	-7
	Regional	28	-23	-28	-20	-16
	All (excl. TEs)	3	-15	-26	-17	-14
Capacity 5 - 50 Mbps	Intercapital	-64	-64	-72	-71	-70
	Metro	-41	-46	-53	-49	-48
	Regional	3	-61	-60	-57	-55
	All (excl. TEs)	-38	-55	-61	-58	-57
Capacity 50 - 200 Mbps	Intercapital	-71	-77	-81	-80	-81
	Metro	-64	-69	-71	-69	-68
	Regional	-33	-78	-78	-76	-75
	All (excl. TEs)	-54	-75	-76	-75	-74
Capacity 200+ Mbps	Intercapital	-92	-92	-95	-95	-95
	Metro	-86	-83	-86	-85	-85
	Regional	-85	-89	-90	-89	-89
	All (excl. TEs)	-89	-89	-92	-91	-91
All capacities	Intercapital	-88	-88	-92	-92	-92
	Metro	-69	-68	-73	-70	-70
	Regional	-58	-79	-80	-78	-77
	All (excl. TEs)	-75	-80	-84	-82	-82
	Tail-end	-62	-44	-58	-54	-53
	All (incl. TEs)	-74	-79	-82	-81	-80

Table A.6: **Model A1: Difference in predicted prices compared to Actual (%)**

Capacity class	*Route type*	*DAA 2015*	*Model (A1a)*	*Model (A1b)*	*Models (A1c)*
Regulated routes					
Capacity < 5 Mbps	Intercapital	-21	-40	-35	-33
	Metro	14	-3	9	13
	Regional	-43	-46	-40	-37
	All (excl. TEs)	-27	-34	-27	-24
Capacity 5 - 50 Mbps	Intercapital	-27	-44	-41	-40
	Metro	-16	-28	-23	-20
	Regional	-66	-65	-62	-60
	All (excl. TEs)	-50	-54	-50	-48
Capacity 50 - 200 Mbps	Intercapital	-64	-70	-68	-69
	Metro	-18	-24	-18	-15
	Regional	-71	-70	-68	-66
	All (excl. TEs)	-62	-63	-60	-59
Capacity 200+ Mbps	Intercapital	-53	-68	-67	-67
	Metro	13	-4	2	6
	Regional	-28	-37	-33	-31
	All (excl. TEs)	-23	-36	-31	-29
All capacities	Intercapital	-50	-62	-60	-60
	Metro	1	-12	-4	-1
	Regional	-54	-56	-52	-50
	All (excl. TEs)	-41	-47	-43	-41
	Tail-end	47	9	20	24
	All (incl. TEs)	-32	-41	-36	-33
Deregulated					
Capacity < 5 Mbps	Intercapital	16	-15	-8	-6
	Metro	7	-10	0	3
	Regional	10	-1	10	13
	All (excl. TEs)	8	-10	0	3
Capacity 5 - 50 Mbps	Intercapital	18	-10	-4	-4
	Metro	-5	-17	-11	-9
	Regional	-21	-22	-16	-12
	All (excl. TEs)	0	-16	-10	-8
Capacity 50 - 200 Mbps	Intercapital	25	0	5	5
	Metro	-9	-16	-10	-8
	Regional	-43	-43	-39	-37
	All (excl. TEs)	-8	-18	-12	-11
Capacity 200+ Mbps	Intercapital	27	-27	-23	-25
	Metro	22	1	8	10
	Regional	-14	-23	-18	-15
	All (excl. TEs)	20	-17	-12	-12
All capacities	Intercapital	25	-20	-15	-16
	Metro	4	-11	-3	-1
	Regional	-21	-27	-21	-19
	All (excl. TEs)	8	-16	-10	-8
	Tail-end	.	.	.	.
	All (incl. TEs)	8	-16	-10	-8

Capacity class	*Route type*	*DAA 2015*	*Model (A1a)*	*Model (A1b)*	*Models (A1c)*
Regulated & Deregulated					
Capacity < 5 Mbps	Intercapital	-4	-28	-22	-20
	Metro	10	-7	4	8
	Regional	-40	-43	-37	-34
	All (excl. TEs)	-17	-28	-20	-17
Capacity 5 - 50 Mbps	Intercapital	2	-22	-17	-17
	Metro	-8	-21	-15	-12
	Regional	-63	-62	-58	-56
	All (excl. TEs)	-27	-36	-32	-30
Capacity 50 - 200 Mbps	Intercapital	-22	-37	-33	-34
	Metro	-12	-19	-13	-11
	Regional	-68	-67	-64	-63
	All (excl. TEs)	-45	-49	-45	-43
Capacity 200+ Mbps	Intercapital	8	-37	-33	-35
	Metro	19	-1	6	9
	Regional	-24	-34	-29	-27
	All (excl. TEs)	1	-25	-20	-19
All capacities	Intercapital	-1	-34	-30	-31
	Metro	3	-11	-4	-1
	Regional	-50	-52	-48	-46
	All (excl. TEs)	-22	-35	-29	-28
	Tail-end	47	9	20	24
	All (incl. TEs)	-17	-32	-26	-24

Table A.7: **Model A1: (2014 data excl. new obs.): Cost elasticities**

	Model (A1a)		**Model (A1b)**		**Model (A1c)**	
	capacity	*distance*	*capacity*	*distance*	*capacity*	*distance*
Deregulated routes						
10th	0.471	0.105	0.466	0.102	0.465	0.121
50th	0.386	0.122	0.382	0.120	0.379	0.136
90th	0.268	0.163	0.267	0.163	0.261	0.175
Regulated routes						
10th	0.468	0.116	0.463	0.113	0.461	0.132
50th	0.421	0.135	0.417	0.134	0.413	0.151
90th	0.294	0.157	0.293	0.157	0.287	0.170

Source: Economic Insights.

A2 Excluding Extreme Observations

Table A.8: **Model A2: RE models excl. influential obs. (2014 data excl. new obs.)**

	Model (A2a)		Model (A2b)		Model (A2c)	
Predictor	*Coeff.*	*t-stat.*	*Coeff.*	*t-stat.*	*Coeff.*	*t-stat.*
constant	5.24297	33.57	4.66257	38.71	4.82920	53.68
log capacity	0.46849	46.33	0.46404	45.93	0.46357	45.93
log distance	0.11217	4.35	0.10982	4.24	0.12897	5.05
0.5(log capacity)^2	-0.02917	-11.29	-0.02845	-11.00	-0.02826	-10.93
0.5(log distance)^2	0.00988	1.44	0.01026	1.49	0.00964	1.39
(log capacity)(log distance)	-0.00111	-0.81	-0.00105	-0.76	-0.00205	-1.52
log route t'put	-0.01898	-3.36	-0.02008	-3.54	.	.
log ESA t'put	0.03106	3.20	0.02854	2.93	.	.
contract start date	-0.00003	-5.79	.	.	.	.
route class 2 (Metro)	0.17848	2.34	0.18043	2.36	0.21766	2.86
route class 3 (Regional)	0.32143	5.45	0.31982	5.40	0.32769	5.50
Provider #1	[redacted]					
Provider #3						
Provider #4						
Provider #5						
Provider #6						
Provider #7						
Provider #8						
Provider #9						
interface-type 3 (SDH)	0.24575	14.78	0.27115	16.87	0.26872	16.72
σ(u)	0.38916		0.39178		0.39486	
σ(e)	0.33138		0.33194		0.33183	

Source: Economic Insights estimation results.

Notes: * Squared correlation between fitted and actual dependent.

Table A.9: **Model A2: RE models excl. influential obs. (2014 data excl. new obs.): Goodness-of-fit**

	Model (A2a)		Model (A2b)		Model (A2c)	
Goodness-of-fit statistic	*10-fold validation (avg)*	*Full sample*	*10-fold validation (avg)*	*Full sample*	*10-fold validation (avg)*	*Full sample*
Within-sample						
obs	5849	6499	5849	6499	5849	6499
R^2*	0.72430	0.72367	0.72329	0.72265	0.71909	0.71865
BIC		6577.2		6601.9		6598.5
RMSE (based on ue)	0.47506	0.47568	0.47588	0.47650	0.47997	0.48046
MAE (based on ue)	0.34248	0.34299	0.34290	0.34344	0.34708	0.34736
Out-of-sample						
obs	677	0	677	0	677	0
R^2*	0.68530	.	0.68360	.	0.67942	.
RMSE (based on ue)	0.53262	.	0.53400	.	0.53823	.
MAE (based on ue)	0.37334	.	0.37396	.	0.37805	.

Notes: * Squared correlation between fitted and actual dependent.

Table A.10: **Model A2: RE models excl. influential obs. (2014 data excl new obs.): Statistical tests**

	Model (A2a)		Model (A2b)		Model (A2c)	
	Stat.	*P-value**	*Stat.*	*P-value**	*Stat.*	*P-value**
Normality of residuals						
Doornik-Hansen(1)	729.1	0.0000	719.2	0.0000	717.8	0.0000
IQR (% severe outliers) (2)†	0.68%		0.67%		0.67%	
Homoscedasticity						
Breusch-Pagan/Cook-Weisberg(6)	120.1	0.0000	130.8	0.0000	141.8	0.0000
Multicollinearity						
# VIF scores > 10	5/19		5/18		5/16	
Misspecification						
RESET (7)	16.2	0.0000	9.0	0.0000	8.7	0.0000
Link test(8)	2.55	0.011	1.94	0.053	1.87	0.061
Joint significance tests						
2nd order output terms (df = 3)	144.9	0.0000	137.6	0.0000	142.0	0.0000
Route classes (df = 2)	40.1	0.0000	38.9	0.0000	36.6	0.0000
Provider fixed effects (df = 7)	1431.5	0.0000	1455.3	0.0000	1453.1	0.0000

Note: * Null hypothesis is rejected, as a standard procedure, in these tests, if P-value is less than 0.05. Equivalently, the reported statistic exceeds the critical value for that statistic; † Percentage of n = 6499 observations; (1) $chi^2(2k)$ where k = 19 for 1st model, and k = 18 for 2nd model and k = 16 for 3rd model. (2) Severe outliers represent about 0.0002% of a normal distribution; (3) Studentized residual > 3; (4) Hat value > $3k/n$; (5) Cook's D > 5 × average Cook's D; (6) $chi^2(1)$; (7) Via powers of the dependent variable, $F(3,n–k–3)$; (8) t-statistic on hat^2; (9) $F(r,n–k–r)$, where r = number of parameters tested, and r = 3 for higher-order output terms, r = 2 for route classes, and r = 8 for provider-specific effects. (10) $chibar^2(1)$; a Approximate, based on OLS regression of $(y - \hat{u}_i)$ on the predictors.

Table A.11: **Model A2: (2014 data excl. new obs.): Cost elasticities**

	Model (A2a)		Model (A2b)		Model (A2c)	
	capacity	*distance*	*capacity*	*distance*	*capacity*	*distance*
Deregulated routes						
10th	0.448	0.117	0.444	0.115	0.443	0.133
50th	0.382	0.134	0.380	0.133	0.378	0.148
90th	0.295	0.171	0.295	0.171	0.289	0.181
Regulated routes						
10th	0.447	0.125	0.443	0.123	0.441	0.141
50th	0.412	0.142	0.409	0.141	0.406	0.157
90th	0.315	0.165	0.314	0.166	0.309	0.176

Source: Economic Insights.

Table A.12: **Model A2: Average predicted prices by market segment ($/month)**

Capacity class	*Route type*	*Actual*	*DAA 2012*	*Model (A2a)*	*Model (A2b)*	*Models (A2c)*
Regulated routes						
Capacity < 5 Mbps	Intercapital	1209	1365	730	775	791
	Metro	375	457	375	403	417
	Regional	1043	781	583	626	651
	All (excl. TEs)	713	635	482	518	537
Capacity 5 - 50 Mbps	Intercapital	2881	6192	1600	1658	1665
	Metro	1075	1678	753	789	821
	Regional	3995	3409	1417	1497	1568
	All (excl. TEs)	2387	2809	1096	1151	1196
Capacity 50 - 200 Mbps	Intercapital	9260	15157	2840	2973	2936
	Metro	2197	6295	1656	1735	1801
	Regional	9691	13374	3022	3170	3288
	All (excl. TEs)	6388	10441	2411	2528	2610
Capacity 200+ Mbps	Intercapital	14561	61066	4982	5186	5075
	Metro	2684	16947	2609	2728	2827
	Regional	7458	48954	5110	5372	5510
	All (excl. TEs)	5611	33806	3842	4028	4127
All capacities	Intercapital	4842	12774	1885	1970	1957
	Metro	782	2321	693	733	760
	Regional	2630	5544	1213	1285	1331
	All (excl. TEs)	1811	4267	985	1041	1076
	Tail-end	[redacted]	1064	445	475	487
	All (incl. TEs)		3115	791	838	864
Deregulated						
Capacity < 5 Mbps	Intercapital	812	1340	691	731	744
	Metro	397	439	363	390	402
	Regional	504	653	513	551	565
	All (excl. TEs)	434	516	398	427	440
Capacity 5 - 50 Mbps	Intercapital	1534	4853	1368	1420	1426
	Metro	859	1490	684	718	736
	Regional	1901	4346	1490	1558	1631
	All (excl. TEs)	1010	2162	828	866	885
Capacity 50 - 200 Mbps	Intercapital	2474	13244	2499	2595	2579
	Metro	1680	4685	1360	1425	1462
	Regional	4432	10605	2579	2699	2791
	All (excl. TEs)	2090	6940	1700	1775	1807
Capacity 200+ Mbps	Intercapital	7744	122355	6458	6752	6561
	Metro	2514	18773	2536	2654	2722
	Regional	5421	37840	4470	4701	4804
	All (excl. TEs)	4418	52098	3935	4120	4114
All capacities	Intercapital	3316	38645	2890	3017	2965
	Metro	895	3073	792	835	858
	Regional	2116	8966	1617	1705	1752
	All (excl. TEs)	1337	8531	1155	1213	1228
	Tail-end	.	.	.	.	.
	All (incl. TEs)	1337	8531	1155	1213	1228

Capacity class	*Route type*	*Actual*	*DAA 2012*	*Model (A2a)*	*Model (A2b)*	*Models (A2c)*
Regulated & Deregulated						
Capacity < 5 Mbps	Intercapital	979	1350	707	749	763
	Metro	387	448	369	396	410
	Regional	980	766	575	617	641
	All (excl. TEs)	606	589	450	483	500
Capacity 5 - 50 Mbps	Intercapital	1842	5160	1421	1474	1480
	Metro	911	1535	700	735	757
	Regional	3675	3552	1428	1506	1578
	All (excl. TEs)	1465	2376	917	960	988
Capacity 50 - 200 Mbps	Intercapital	4032	13683	2577	2682	2661
	Metro	1835	5169	1449	1518	1564
	Regional	8561	12778	2927	3069	3181
	All (excl. TEs)	3857	8379	1992	2085	2137
Capacity 200+ Mbps	Intercapital	8686	113887	6254	6536	6356
	Metro	2573	18139	2561	2680	2758
	Regional	6838	45569	4915	5167	5295
	All (excl. TEs)	4874	45105	3900	4085	4119
All capacities	Intercapital	3711	31948	2630	2746	2704
	Metro	849	2764	751	793	818
	Regional	2555	6047	1272	1346	1393
	All (excl. TEs)	1584	6310	1066	1124	1148
	Tail-end	[redacted]	1064	445	475	487
	All (incl. TEs)	[redacted]	5124	926	977	999

Table A.13: **Model A2: Difference in predicted prices compared to DAA 2012 (%)**

Capacity class	*Route type*	*Model (A2a)*	*Model (A2b)*	*Models (A2c)*
Regulated routes				
Capacity < 5 Mbps	Intercapital	-47	-43	-42
	Metro	-18	-12	-9
	Regional	-25	-20	-17
	All (excl. TEs)	-24	-18	-15
Capacity 5 - 50 Mbps	Intercapital	-74	-73	-73
	Metro	-55	-53	-51
	Regional	-58	-56	-54
	All (excl. TEs)	-61	-59	-57
Capacity 50 - 200 Mbps	Intercapital	-81	-80	-81
	Metro	-74	-72	-71
	Regional	-77	-76	-75
	All (excl. TEs)	-77	-76	-75
Capacity 200+ Mbps	Intercapital	-92	-92	-92
	Metro	-85	-84	-83
	Regional	-90	-89	-89
	All (excl. TEs)	-89	-88	-88
All capacities	Intercapital	-85	-85	-85
	Metro	-70	-68	-67
	Regional	-78	-77	-76
	All (excl. TEs)	-77	-76	-75

Capacity class	*Route type*	*Model (A2a)*	*Model (A2b)*	*Models (A2c)*
	Tail-end	-58	-55	-54
	All (incl. TEs)	-75	-73	-72
Deregulated				
Capacity < 5 Mbps	Intercapital	-48	-45	-45
	Metro	-17	-11	-8
	Regional	-21	-16	-13
	All (excl. TEs)	-23	-17	-15
Capacity 5 - 50 Mbps	Intercapital	-72	-71	-71
	Metro	-54	-52	-51
	Regional	-66	-64	-62
	All (excl. TEs)	-62	-60	-59
Capacity 50 - 200 Mbps	Intercapital	-81	-80	-81
	Metro	-71	-70	-69
	Regional	-76	-75	-74
	All (excl. TEs)	-76	-74	-74
Capacity 200+ Mbps	Intercapital	-95	-94	-95
	Metro	-86	-86	-86
	Regional	-88	-88	-87
	All (excl. TEs)	-92	-92	-92
All capacities	Intercapital	-93	-92	-92
	Metro	-74	-73	-72
	Regional	-82	-81	-80
	All (excl. TEs)	-86	-86	-86
	Tail-end	.	.	.
	All (incl. TEs)	-86	-86	-86
Regulated & Deregulated				
Capacity < 5 Mbps	Intercapital	-48	-45	-43
	Metro	-18	-12	-9
	Regional	-25	-19	-16
	All (excl. TEs)	-24	-18	-15
Capacity 5 - 50 Mbps	Intercapital	-72	-71	-71
	Metro	-54	-52	-51
	Regional	-60	-58	-56
	All (excl. TEs)	-61	-60	-58
Capacity 50 - 200 Mbps	Intercapital	-81	-80	-81
	Metro	-72	-71	-70
	Regional	-77	-76	-75
	All (excl. TEs)	-76	-75	-74
Capacity 200+ Mbps	Intercapital	-95	-94	-94
	Metro	-86	-85	-85
	Regional	-89	-89	-88
	All (excl. TEs)	-91	-91	-91
All capacities	Intercapital	-92	-91	-92
	Metro	-73	-71	-70
	Regional	-79	-78	-77
	All (excl. TEs)	-83	-82	-82
	Tail-end	-58	-55	-54
	All (incl. TEs)	-82	-81	-81

Table A.14: **Model A2: Difference in predicted prices compared to actual (%)**

Capacity class	*Route type*	*DAA 2012*	*Model (A2a)*	*Model (A2b)*	*Models (A2c)*
Regulated routes					
Capacity < 5 Mbps	Intercapital	13	-40	-36	-35
	Metro	22	0	7	11
	Regional	-25	-44	-40	-38
	All (excl. TEs)	-11	-32	-27	-25
Capacity 5 - 50 Mbps	Intercapital	115	-44	-42	-42
	Metro	56	-30	-27	-24
	Regional	-15	-65	-63	-61
	All (excl. TEs)	18	-54	-52	-50
Capacity 50 - 200 Mbps	Intercapital	64	-69	-68	-68
	Metro	186	-25	-21	-18
	Regional	38	-69	-67	-66
	All (excl. TEs)	63	-62	-60	-59
Capacity 200+ Mbps	Intercapital	319	-66	-64	-65
	Metro	531	-3	2	5
	Regional	556	-31	-28	-26
	All (excl. TEs)	502	-32	-28	-26
All capacities	Intercapital	164	-61	-59	-60
	Metro	197	-11	-6	-3
	Regional	111	-54	-51	-49
	All (excl. TEs)	136	-46	-42	-41
	Tail-end	160	9	16	19
	All (incl. TEs)	138	-40	-36	-34
Deregulated					
Capacity < 5 Mbps	Intercapital	65	-15	-10	-8
	Metro	11	-9	-2	1
	Regional	29	2	9	12
	All (excl. TEs)	19	-8	-1	1
Capacity 5 - 50 Mbps	Intercapital	216	-11	-7	-7
	Metro	73	-20	-17	-14
	Regional	129	-22	-18	-14
	All (excl. TEs)	114	-18	-14	-12
Capacity 50 - 200 Mbps	Intercapital	435	1	5	4
	Metro	179	-19	-15	-13
	Regional	139	-42	-39	-37
	All (excl. TEs)	232	-19	-15	-14
Capacity 200+ Mbps	Intercapital	1480	-17	-13	-15
	Metro	647	1	6	8
	Regional	598	-18	-13	-11
	All (excl. TEs)	1079	-11	-7	-7
All capacities	Intercapital	1065	-13	-9	-11
	Metro	243	-12	-7	-4
	Regional	324	-24	-19	-17
	All (excl. TEs)	538	-14	-9	-8
	Tail-end	.	.	.	.
	All (incl. TEs)	538	-14	-9	-8

Capacity class	*Route type*	*DAA 2012*	*Model (A2a)*	*Model (A2b)*	*Models (A2c)*
Regulated & Deregulated					
Capacity < 5 Mbps	Intercapital	38	-28	-23	-22
	Metro	16	-5	3	6
	Regional	-22	-41	-37	-35
	All (excl. TEs)	-3	-26	-20	-18
Capacity 5 - 50 Mbps	Intercapital	180	-23	-20	-20
	Metro	68	-23	-19	-17
	Regional	-3	-61	-59	-57
	All (excl. TEs)	62	-37	-34	-33
Capacity 50 - 200 Mbps	Intercapital	239	-36	-33	-34
	Metro	182	-21	-17	-15
	Regional	49	-66	-64	-63
	All (excl. TEs)	117	-48	-46	-45
Capacity 200+ Mbps	Intercapital	1211	-28	-25	-27
	Metro	605	0	4	7
	Regional	566	-28	-24	-23
	All (excl. TEs)	825	-20	-16	-15
All capacities	Intercapital	761	-29	-26	-27
	Metro	226	-11	-7	-4
	Regional	137	-50	-47	-45
	All (excl. TEs)	298	-33	-29	-27
	Tail-end	160	9	16	19
	All (incl. TEs)	289	-30	-26	-24

Source: Economic Insights estimates.

A3 Robust RE

Table A.15: **Model 3: Robust RE models (2014 data excl. new obs.)**

	Model (A3a)		Model (A3b)		Model (A3c)	
Predictor	*Coeff.*	*t-stat.*	*Coeff.*	*t-stat.*	*Coeff.*	*t-stat.*
constant	0.01385	1.58	0.01315	1.52	0.00843	0.98
log capacity	0.44723	44.29	0.44021	43.62	0.44359	43.93
log distance	0.10540	6.85	0.10387	6.83	0.12102	7.64
0.5(log capacity)^2	-0.02179	-8.65	-0.02059	-8.17	-0.02144	-8.49
0.5(log distance)^2	0.00853	1.91	0.00871	1.97	0.01024	2.20
(log capacity)(log distance)	-0.00333	-2.58	-0.00319	-2.47	-0.00391	-3.08
log route t'put	-0.01387	-3.56	-0.01503	-3.89		
log ESA t'put	0.04878	7.39	0.04787	7.31		
contract start date	-0.00003	-5.45				
route class 2 (Metro)	0.12025	2.28	0.12377	2.36	0.17998	3.34
route class 3 (Regional)	0.24976	5.84	0.25176	5.91	0.25217	5.73
Provider #1						
Provider #3						
Provider #4						
Provider #5						
Provider #6						
Provider #7						
Provider #8						
Provider #9						
interface-type 3 (SDH)	0.23812	14.10	0.25945	15.95	0.25497	15.65
σ(u)	0.15980		0.15570		0.17760	
σ(e)	0.38150		0.38300		0.38010	
Required intercept adjustment	5.10831		4.52345		4.89958	
Intercept after adjustment	5.12216		4.53660		4.90801	
Goodness-of-fit						
R^2*	0.68676		0.68519		0.67967	
RMSE (based on ue)	0.53143		0.53270		0.53764	
MAE (based on ue)	0.36652		0.36679		0.37494	

Source: Economic Insights estimation results.
Notes: * Squared correlation between fitted and actual dependent.

Table A.16: **Model A2: (2014 data excl. new obs.): Cost elasticities**

	Model (A3a)		Model (A3b)		Model (A3c)	
	capacity	*distance*	*capacity*	*distance*	*capacity*	*distance*
Deregulated routes						
10th	0.430	0.108	0.424	0.106	0.427	0.124
50th	0.377	0.118	0.373	0.117	0.372	0.136
90th	0.301	0.142	0.302	0.143	0.296	0.166
Regulated routes						
10th	0.427	0.115	0.421	0.114	0.423	0.133
50th	0.397	0.127	0.393	0.127	0.392	0.147
90th	0.318	0.139	0.317	0.140	0.312	0.162

Source: Economic Insights.

Table A17: **Model 3: Average predicted prices by market segment ($/month)**

Capacity class	*Route type*	*Actual*	*DAA 2012*	*Model (A3a)*	*Model (A3b)*	*Models (A3c)*
Regulated routes						
Capacity < 5 Mbps	Intercapital	1209	1365	741	785	802
	Metro	375	457	376	403	421
	Regional	1043	781	549	588	623
	All (excl. TEs)	713	635	468	501	527
Capacity 5 - 50 Mbps	Intercapital	2881	6192	1550	1603	1630
	Metro	1075	1678	735	768	814
	Regional	3995	3409	1264	1331	1445
	All (excl. TEs)	2387	2809	1023	1071	1141
Capacity 50 - 200 Mbps	Intercapital	9260	15157	2745	2863	2846
	Metro	2197	6295	1612	1680	1780
	Regional	9691	13374	2667	2787	3006
	All (excl. TEs)	6388	10441	2214	2311	2458
Capacity 200+ Mbps	Intercapital	14561	61066	4866	5059	5002
	Metro	2684	16947	2592	2698	2849
	Regional	7458	48954	4649	4877	5181
	All (excl. TEs)	5611	33806	3632	3796	3994
All capacities	Intercapital	4842	12774	1844	1924	1927
	Metro	782	2321	687	724	762
	Regional	2630	5544	1107	1169	1248
	All (excl. TEs)	1811	4267	931	982	1037
	Tail-end	[redacted]	1064	447	475	491
	All (incl. TEs)	[redacted]	3115	757	800	841
Deregulated						
Capacity < 5 Mbps	Intercapital	812	1340	704	744	756
	Metro	397	439	365	391	407
	Regional	504	653	489	524	544
	All (excl. TEs)	434	516	399	426	443
Capacity 5 - 50 Mbps	Intercapital	1534	4853	1337	1385	1403
	Metro	859	1490	677	708	738
	Regional	1901	4346	1325	1381	1503
	All (excl. TEs)	1010	2162	812	847	878
Capacity 50 - 200 Mbps	Intercapital	2474	13244	2401	2487	2501
	Metro	1680	4685	1347	1403	1464
	Regional	4432	10605	2302	2402	2570
	All (excl. TEs)	2090	6940	1646	1712	1773
Capacity 200+ Mbps	Intercapital	7744	122355	6466	6765	6620
	Metro	2514	18773	2601	2710	2817
	Regional	5421	37840	4106	4306	4531
	All (excl. TEs)	4418	52098	3935	4111	4155
All capacities	Intercapital	3316	38645	2865	2989	2960
	Metro	895	3073	796	836	870
	Regional	2116	8966	1481	1557	1645
	All (excl. TEs)	1337	8531	1143	1197	1228
	Tail-end	.	.	.	.	.
	All (incl. TEs)	1337	8531	1143	1197	1228

Capacity class	*Route type*	*Actual*	*DAA 2012*	*Model (A3a)*	*Model (A3b)*	*Models (A3c)*
Regulated & Deregulated						
Capacity < 5 Mbps	Intercapital	979	1350	720	761	776
	Metro	387	448	371	397	414
	Regional	980	766	542	581	614
	All (excl. TEs)	606	589	441	472	495
Capacity 5 - 50 Mbps	Intercapital	1842	5160	1386	1435	1455
	Metro	911	1535	691	722	756
	Regional	3675	3552	1273	1338	1454
	All (excl. TEs)	1465	2376	882	921	965
Capacity 50 - 200 Mbps	Intercapital	4032	13683	2480	2574	2580
	Metro	1835	5169	1427	1486	1559
	Regional	8561	12778	2589	2704	2912
	All (excl. TEs)	3857	8379	1879	1958	2055
Capacity 200+ Mbps	Intercapital	8686	113887	6245	6529	6396
	Metro	2573	18139	2598	2706	2828
	Regional	6838	45569	4484	4703	4983
	All (excl. TEs)	4874	45105	3819	3991	4094
All capacities	Intercapital	3711	31948	2601	2713	2692
	Metro	849	2764	752	790	826
	Regional	2555	6047	1162	1226	1307
	All (excl. TEs)	1584	6310	1033	1085	1129
	Tail-end	[redacted]	1064	447	475	491
	All (incl. TEs)		5124	900	947	984

Table A.18: **Model 3: Difference in predicted prices compared to DAA 2012 (%)**

Capacity class	*Route type*	*Model (A3a)*	*Model (A3b)*	*Models (A3c)*
Regulated routes				
Capacity < 5 Mbps	Intercapital	-46	-42	-41
	Metro	-18	-12	-8
	Regional	-30	-25	-20
	All (excl. TEs)	-26	-21	-17
Capacity 5 - 50 Mbps	Intercapital	-75	-74	-74
	Metro	-56	-54	-51
	Regional	-63	-61	-58
	All (excl. TEs)	-64	-62	-59
Capacity 50 - 200 Mbps	Intercapital	-82	-81	-81
	Metro	-74	-73	-72
	Regional	-80	-79	-78
	All (excl. TEs)	-79	-78	-76
Capacity 200+ Mbps	Intercapital	-92	-92	-92
	Metro	-85	-84	-83
	Regional	-91	-90	-89
	All (excl. TEs)	-89	-89	-88
All capacities	Intercapital	-86	-85	-85
	Metro	-70	-69	-67
	Regional	-80	-79	-77
	All (excl. TEs)	-78	-77	-76
	Tail-end	-58	-55	-54

Capacity class	*Route type*	*Model (A3a)*	*Model (A3b)*	*Models (A3c)*
	All (incl. TEs)	-76	-74	-73
Deregulated				
Capacity < 5 Mbps	Intercapital	-47	-44	-44
	Metro	-17	-11	-7
	Regional	-25	-20	-17
	All (excl. TEs)	-23	-17	-14
Capacity 5 - 50 Mbps	Intercapital	-72	-71	-71
	Metro	-55	-52	-50
	Regional	-70	-68	-65
	All (excl. TEs)	-62	-61	-59
Capacity 50 - 200 Mbps	Intercapital	-82	-81	-81
	Metro	-71	-70	-69
	Regional	-78	-77	-76
	All (excl. TEs)	-76	-75	-74
Capacity 200+ Mbps	Intercapital	-95	-94	-95
	Metro	-86	-86	-85
	Regional	-89	-89	-88
	All (excl. TEs)	-92	-92	-92
All capacities	Intercapital	-93	-92	-92
	Metro	-74	-73	-72
	Regional	-83	-83	-82
	All (excl. TEs)	-87	-86	-86
	Tail-end	.	.	.
	All (incl. TEs)	-87	-86	-86
Regulated & Deregulated				
Capacity < 5 Mbps	Intercapital	-47	-44	-43
	Metro	-17	-11	-8
	Regional	-29	-24	-20
	All (excl. TEs)	-25	-20	-16
Capacity 5 - 50 Mbps	Intercapital	-73	-72	-72
	Metro	-55	-53	-51
	Regional	-64	-62	-59
	All (excl. TEs)	-63	-61	-59
Capacity 50 - 200 Mbps	Intercapital	-82	-81	-81
	Metro	-72	-71	-70
	Regional	-80	-79	-77
	All (excl. TEs)	-78	-77	-75
Capacity 200+ Mbps	Intercapital	-95	-94	-94
	Metro	-86	-85	-84
	Regional	-90	-90	-89
	All (excl. TEs)	-92	-91	-91
All capacities	Intercapital	-92	-92	-92
	Metro	-73	-71	-70
	Regional	-81	-80	-78
	All (excl. TEs)	-84	-83	-82
	Tail-end	-58	-55	-54
	All (incl. TEs)	-82	-82	-81

Table A.19: **Model 3: Difference in predicted prices compared to actual (%)**

Capacity class	*Route type*	*DAA 2012*	*Model (A3a)*	*Model (A3b)*	*Models (A3c)*
Regulated routes					
Capacity < 5 Mbps	Intercapital	13	-39	-35	-34
	Metro	22	0	7	12
	Regional	-25	-47	-44	-40
	All (excl. TEs)	-11	-34	-30	-26
Capacity 5 - 50 Mbps	Intercapital	115	-46	-44	-43
	Metro	56	-32	-29	-24
	Regional	-15	-68	-67	-64
	All (excl. TEs)	18	-57	-55	-52
Capacity 50 - 200 Mbps	Intercapital	64	-70	-69	-69
	Metro	186	-27	-24	-19
	Regional	38	-72	-71	-69
	All (excl. TEs)	63	-65	-64	-62
Capacity 200+ Mbps	Intercapital	319	-67	-65	-66
	Metro	531	-3	1	6
	Regional	556	-38	-35	-31
	All (excl. TEs)	502	-35	-32	-29
All capacities	Intercapital	164	-62	-60	-60
	Metro	197	-12	-7	-2
	Regional	111	-58	-56	-53
	All (excl. TEs)	136	-49	-46	-43
	Tail-end	160	9	16	20
	All (incl. TEs)	138	-42	-39	-36
Deregulated					
Capacity < 5 Mbps	Intercapital	65	-13	-8	-7
	Metro	11	-8	-2	3
	Regional	29	-3	4	8
	All (excl. TEs)	19	-8	-2	2
Capacity 5 - 50 Mbps	Intercapital	216	-13	-10	-9
	Metro	73	-21	-18	-14
	Regional	129	-30	-27	-21
	All (excl. TEs)	114	-20	-16	-13
Capacity 50 - 200 Mbps	Intercapital	435	-3	1	1
	Metro	179	-20	-16	-13
	Regional	139	-48	-46	-42
	All (excl. TEs)	232	-21	-18	-15
Capacity 200+ Mbps	Intercapital	1480	-17	-13	-15
	Metro	647	3	8	12
	Regional	598	-24	-21	-16
	All (excl. TEs)	1079	-11	-7	-6
All capacities	Intercapital	1065	-14	-10	-11
	Metro	243	-11	-7	-3
	Regional	324	-30	-26	-22
	All (excl. TEs)	538	-15	-10	-8
	Tail-end	.	.	.	.
	All (incl. TEs)	538	-15	-10	-8

Capacity class	*Route type*	*DAA 2012*	*Model (A3a)*	*Model (A3b)*	*Models (A3c)*
Regulated & Deregulated					
Capacity < 5 Mbps	Intercapital	38	-26	-22	-21
	Metro	16	-4	3	7
	Regional	-22	-45	-41	-37
	All (excl. TEs)	-3	-27	-22	-18
Capacity 5 - 50 Mbps	Intercapital	180	-25	-22	-21
	Metro	68	-24	-21	-17
	Regional	-3	-65	-64	-60
	All (excl. TEs)	62	-40	-37	-34
Capacity 50 - 200 Mbps	Intercapital	239	-38	-36	-36
	Metro	182	-22	-19	-15
	Regional	49	-70	-68	-66
	All (excl. TEs)	117	-51	-49	-47
Capacity 200+ Mbps	Intercapital	1211	-28	-25	-26
	Metro	605	1	5	10
	Regional	566	-34	-31	-27
	All (excl. TEs)	825	-22	-18	-16
All capacities	Intercapital	761	-30	-27	-27
	Metro	226	-11	-7	-3
	Regional	137	-55	-52	-49
	All (excl. TEs)	298	-35	-31	-29
	Tail-end	160	9	16	20
	All (incl. TEs)	289	-32	-28	-25

Source: Economic Insights estimates.

A4 Excluding 2.5 Mbps Services

Table A.20: **Model A4: RE models excl. 2 Mbps services* (2014 data excl. new obs.)**

	Model (A4a)		Model (A4b)		Model (A4c)	
Predictor	*Coeff.*	*t-stat.*	*Coeff.*	*t-stat.*	*Coeff.*	*t-stat.*
constant	6.98647	33.53	5.24050	36.69	5.19992	52.67
log capacity	0.49615	37.11	0.48766	36.11	0.48284	35.95
log distance	-0.00355	-0.13	-0.00347	-0.12	0.02006	0.73
0.5(log capacity)^2	-0.03686	-11.80	-0.03569	-11.30	-0.03492	-11.08
0.5(log distance)^2	0.03150	4.46	0.03158	4.43	0.02920	4.09
(log capacity)(log distance)	-0.00172	-1.00	-0.00192	-1.11	-0.00313	-1.84
log route t'put	-0.02072	-3.45	-0.02272	-3.76	.	.
log ESA t'put	0.02294	2.04	0.01018	0.90	.	.
contract start date	-0.00010	-11.41	.	.	.	.
route class 2 (Metro)	0.11153	1.43	0.11894	1.51	0.14815	1.88
route class 3 (Regional)	0.29025	4.80	0.28348	4.65	0.29801	4.86
Provider #1						
Provider #3						
Provider #4						
Provider #5						
Provider #6						
Provider #7						
Provider #8						
Provider #9						
interface-type 3 (SDH)	0.17031	7.65	0.24442	11.35	0.24490	11.40
σ(u)	0.31909		0.32097		0.32633	
σ(e)	0.45403		0.45984		0.45942	

Source: Economic Insights estimation results.

Notes: * Services < 2.5 Mbps & < 5 km.

Table A.21: **Model A4: Base case RE models (2014 data excl. new obs.): Cost elasticities**

	Model (A4a)		Model (A4b)		Model (A4c)	
	capacity	*distance*	*capacity*	*distance*	*capacity*	*distance*
Deregulated routes						
10th	0.470	0.012	0.462	0.012	0.457	0.034
50th	0.387	0.071	0.381	0.071	0.375	0.085
90th	0.275	0.193	0.271	0.193	0.263	0.194
Regulated routes						
10th	0.468	0.039	0.460	0.039	0.454	0.059
50th	0.424	0.096	0.417	0.096	0.410	0.110
90th	0.300	0.175	0.296	0.175	0.288	0.178

Source: Economic Insights.

Table A.22: **Model A4: RE models excl. 2 Mbps services* (2014 data excl. new obs.): Goodness-of-fit**

Goodness-of-fit statistic	**Model (A4a)** *10-fold validation (avg)*	*Full sample*	**Model (A4b)** *10-fold validation (avg)*	*Full sample*	**Model (A4c)** *10-fold validation (avg)*	*Full sample*
Within-sample						
Obs**	6090	6767	6090	6767	6090	6767
R^2*	0.70287	0.70246	0.69862	0.69822	0.69508	0.69478
BIC		8204.6		8324.7		8322.5
RMSE (based on ue)	0.51700	0.51742	0.52051	0.52091	0.52488	0.52529
MAE (based on ue)	0.36835	0.36841	0.36889	0.36898	0.37052	0.37058
Out-of-sample						
Obs**	677	0	677	0	677	0
R^2*	0.70098		0.69670		0.69343	
RMSE (based on ue)	0.51879		0.52226		0.52635	
MAE (based on ue)	0.36967		0.37009		0.37165	

Notes: * Squared correlation between fitted and actual dependent. ** Goodness-of-Fit statistics combine the predictions of the econometric model, used for services ≥ 2.5 Mbps or ≥ 5 km, and the predictor used for services less than both these thresholds (i.e. mean of log monthly price on deregulated routes for services < 2.5 Mbps and < 5 km). There are 5452 observations of the first kind, and 1315 observations of the second kind in the full sample. In the econometric validation samples there are 545 observations and 4907 observations in the corresponding estimation samples.

Table A.23: **Model A4: (2014 data excl. new obs.): Statistical tests**

	Model (A4a) *Stat.*	*P-value**	**Model (A4b)** *Stat.*	*P-value**	**Model (A4c)** *Stat.*	*P-value**
Normality of residuals						
Doornik-Hansen(1)	2930.1	0.0000	3006.5	0.0000	2966.7	0.0000
IQR (% severe outliers) (2)†	1.45%		1.44%		1.34%	
Homoscedasticity						
Breusch-Pagan/Cook-Weisberg(6)	858.8	0.0000	891.8	0.0000	983.3	0.0000
Multicollinearity						
# VIF scores > 10	5/19		5/18		5/16	
Misspecification						
RESET (7)	18.2	0.0000	11.5	0.0000	11.9	0.0000
Link test(8)	2.81	0.005	0.62	0.532	0.37	0.713
Joint significance tests						
2nd order output terms (df = 3)	176.7	0.0000	164.7	0.0000	161.7	0.0000
Route classes (df = 2)	37.5	0.0000	33.7	0.0000	33.8	0.0000
Provider fixed effects (df = 7)	915.4	0.0000	933.7	0.0000	932.9	0.0000

Note: * Null hypothesis is rejected, as a standard procedure, in these tests, if P-value is less than 0.05. Equivalently, the reported statistic exceeds the critical value for that statistic; † Percentage of n = 5452 observations; (1) $chi^2(2k)$ where k = 19 for 1st model, and k = 18 for 2nd model and k = 16 for 3rd model. (2) Severe outliers represent about 0.0002% of a normal distribution; (3) Studentized residual > 3; (4) Hat value > $3k/n$; (5) Cook's D > 5 × average Cook's D; (6) $chi^2(1)$; (7) Via powers of the dependent variable, $F(3,n–k–3)$; (8) t-statistic on hat^2; (9) $F(r,n–k–r)$, where r = number of parameters tested, and r = 3 for higher-order output terms, r = 2 for route classes, and r = 8 for provider-specific effects. (10) $chibar^2(1)$; a Approximate, based on OLS regression of $(y - \hat{u}_i)$ on the predictors.

Table A.24: **Model A4: Average predicted prices by market segment ($/month)**

Capacity class	*Route type*	*Actual*	*DAA 2012*	*Model (A4a)*	*Model (A4b)*	*Models (A4c)*
Regulated routes						
Capacity 5 Mbps*	Intercapital	1209	1365	714	852	873
	Metro	375	457	365	428	436
	Regional	1043	781	548	670	683
	All (excl. TEs)	713	635	460	553	564
Capacity 5 - 50 Mbps	Intercapital	2881	6192	1658	1844	1835
	Metro	1075	1678	757	873	889
	Regional	3995	3409	1383	1642	1658
	All (excl. TEs)	2387	2809	1091	1269	1283
Capacity 50 - 200 Mbps	Intercapital	9260	15157	2798	3202	3134
	Metro	2197	6295	1655	1915	1933
	Regional	9691	13374	2943	3418	3395
	All (excl. TEs)	6388	10441	2369	2745	2735
Capacity 200+ Mbps	Intercapital	14561	61066	4664	5209	5016
	Metro	2684	16947	2498	2864	2878
	Regional	7458	48954	4754	5520	5406
	All (excl. TEs)	5611	33806	3613	4160	4104
All capacities	Intercapital	4842	12774	1843	2095	2065
	Metro	782	2321	678	786	797
	Regional	2630	5544	1152	1366	1367
	All (excl. TEs)	1811	4267	947	1111	1115
	Tail-end	[redacted]	1064	523	633	633
	All (incl. TEs)	[redacted]	3115	795	939	942
Deregulated						
Capacity 5 Mbps*	Intercapital	812	1340	669	790	808
	Metro	397	439	365	430	437
	Regional	504	653	480	575	580
	All (excl. TEs)	434	516	396	467	474
Capacity 5 - 50 Mbps	Intercapital	1534	4853	1384	1548	1546
	Metro	859	1490	701	802	805
	Regional	1901	4346	1459	1676	1688
	All (excl. TEs)	1010	2162	843	960	962
Capacity 50 - 200 Mbps	Intercapital	2474	13244	2542	2840	2790
	Metro	1680	4685	1334	1530	1528
	Regional	4432	10605	2470	2852	2833
	All (excl. TEs)	2090	6940	1680	1913	1900
Capacity 200+ Mbps	Intercapital	7744	122355	5883	6600	6305
	Metro	2514	18773	2400	2747	2733
	Regional	5421	37840	4195	4889	4792
	All (excl. TEs)	4418	52098	3652	4149	4042
All capacities	Intercapital	3316	38645	2740	3078	2989
	Metro	895	3073	781	900	902
	Regional	2116	8966	1531	1787	1771
	All (excl. TEs)	1337	8531	1118	1279	1267
	Tail-end	.	.	.	.	.
	All (incl. TEs)	1337	8531	1118	1279	1267

Capacity class	*Route type*	*Actual*	*DAA 2012*	*Model (A4a)*	*Model (A4b)*	*Models (A4c)*
Regulated & Deregulated						
Capacity 5 Mbps*	Intercapital	979	1350	688	816	835
	Metro	387	448	365	429	436
	Regional	980	766	540	659	671
	All (excl. TEs)	606	589	436	520	530
Capacity 5 - 50 Mbps	Intercapital	1842	5160	1447	1616	1612
	Metro	911	1535	714	819	825
	Regional	3675	3552	1395	1647	1663
	All (excl. TEs)	1465	2376	925	1062	1068
Capacity 50 - 200 Mbps	Intercapital	4032	13683	2601	2923	2869
	Metro	1835	5169	1431	1645	1649
	Regional	8561	12778	2841	3297	3275
	All (excl. TEs)	3857	8379	1963	2255	2243
Capacity 200+ Mbps	Intercapital	8686	113887	5715	6408	6127
	Metro	2573	18139	2434	2787	2783
	Regional	6838	45569	4584	5328	5219
	All (excl. TEs)	4874	45105	3637	4153	4066
All capacities	Intercapital	3711	31948	2508	2824	2750
	Metro	849	2764	738	853	859
	Regional	2555	6047	1208	1428	1426
	All (excl. TEs)	1584	6310	1029	1192	1188
	Tail-end	[redacted]	1064	523	633	633
	All (incl. TEs)	[redacted]	5124	915	1065	1063

* Excludes services < 2.5 Mbps and < 50 km.

Table A.25: **Model 4: Difference in predicted prices compared to DAA 2012 (%)**

Capacity class	*Route type*	*Model (A4a)*	*Model (A4b)*	*Models (A4c)*
Regulated routes				
Capacity < 5 Mbps	Intercapital	-48	-38	-36
	Metro	-20	-6	-5
	Regional	-30	-14	-13
	All (excl. TEs)	-27	-13	-11
Capacity 5 - 50 Mbps	Intercapital	-73	-70	-70
	Metro	-55	-48	-47
	Regional	-59	-52	-51
	All (excl. TEs)	-61	-55	-54
Capacity 50 - 200 Mbps	Intercapital	-82	-79	-79
	Metro	-74	-70	-69
	Regional	-78	-74	-75
	All (excl. TEs)	-77	-74	-74
Capacity 200+ Mbps	Intercapital	-92	-91	-92
	Metro	-85	-83	-83
	Regional	-90	-89	-89
	All (excl. TEs)	-89	-88	-88
All capacities	Intercapital	-86	-84	-84
	Metro	-71	-66	-66
	Regional	-79	-75	-75
	All (excl. TEs)	-78	-74	-74

Capacity class	*Route type*	*Model (A4a)*	*Model (A4b)*	*Models (A4c)*
	Tail-end	-51	-41	-41
	All (incl. TEs)	-74	-70	-70
Deregulated				
Capacity < 5 Mbps	Intercapital	-50	-41	-40
	Metro	-17	-2	-1
	Regional	-26	-12	-11
	All (excl. TEs)	-23	-10	-8
Capacity 5 - 50 Mbps	Intercapital	-71	-68	-68
	Metro	-53	-46	-46
	Regional	-66	-61	-61
	All (excl. TEs)	-61	-56	-56
Capacity 50 - 200 Mbps	Intercapital	-81	-79	-79
	Metro	-72	-67	-67
	Regional	-77	-73	-73
	All (excl. TEs)	-76	-72	-73
Capacity 200+ Mbps	Intercapital	-95	-95	-95
	Metro	-87	-85	-85
	Regional	-89	-87	-87
	All (excl. TEs)	-93	-92	-92
All capacities	Intercapital	-93	-92	-92
	Metro	-75	-71	-71
	Regional	-83	-80	-80
	All (excl. TEs)	-87	-85	-85
	Tail-end	.	.	.
	All (incl. TEs)	-87	-85	-85
Regulated & Deregulated				
Capacity < 5 Mbps	Intercapital	-49	-40	-38
	Metro	-19	-4	-3
	Regional	-30	-14	-12
	All (excl. TEs)	-26	-12	-10
Capacity 5 - 50 Mbps	Intercapital	-72	-69	-69
	Metro	-53	-47	-46
	Regional	-61	-54	-53
	All (excl. TEs)	-61	-55	-55
Capacity 50 - 200 Mbps	Intercapital	-81	-79	-79
	Metro	-72	-68	-68
	Regional	-78	-74	-74
	All (excl. TEs)	-77	-73	-73
Capacity 200+ Mbps	Intercapital	-95	-94	-95
	Metro	-87	-85	-85
	Regional	-90	-88	-89
	All (excl. TEs)	-92	-91	-91
All capacities	Intercapital	-92	-91	-91
	Metro	-73	-69	-69
	Regional	-80	-76	-76
	All (excl. TEs)	-84	-81	-81
	Tail-end	-51	-41	-41
	All (incl. TEs)	-82	-79	-79

Table A.26: **Model A4: Difference in predicted prices compared to Actual (%)**

Capacity class	*Route type*	*DAA 2012*	*Model (A4a)*	*Model (A4b)*	*Models (A4c)*
Regulated routes					
Capacity < 5 Mbps	Intercapital	13	-41	-30	-28
	Metro	22	-3	14	16
	Regional	-25	-47	-36	-35
	All (excl. TEs)	-11	-35	-22	-21
Capacity 5 - 50 Mbps	Intercapital	115	-42	-36	-36
	Metro	56	-30	-19	-17
	Regional	-15	-65	-59	-58
	All (excl. TEs)	18	-54	-47	-46
Capacity 50 - 200 Mbps	Intercapital	64	-70	-65	-66
	Metro	186	-25	-13	-12
	Regional	38	-70	-65	-65
	All (excl. TEs)	63	-63	-57	-57
Capacity 200+ Mbps	Intercapital	319	-68	-64	-66
	Metro	531	-7	7	7
	Regional	556	-36	-26	-28
	All (excl. TEs)	502	-36	-26	-27
All capacities	Intercapital	164	-62	-57	-57
	Metro	197	-13	1	2
	Regional	111	-56	-48	-48
	All (excl. TEs)	136	-48	-39	-38
	Tail-end	160	28	55	55
	All (incl. TEs)	138	-39	-28	-28
Deregulated					
Capacity < 5 Mbps	Intercapital	65	-18	-3	0
	Metro	11	-8	8	10
	Regional	29	-5	14	15
	All (excl. TEs)	19	-9	8	9
Capacity 5 - 50 Mbps	Intercapital	216	-10	1	1
	Metro	73	-18	-7	-6
	Regional	129	-23	-12	-11
	All (excl. TEs)	114	-17	-5	-5
Capacity 50 - 200 Mbps	Intercapital	435	3	15	13
	Metro	179	-21	-9	-9
	Regional	139	-44	-36	-36
	All (excl. TEs)	232	-20	-8	-9
Capacity 200+ Mbps	Intercapital	1480	-24	-15	-19
	Metro	647	-5	9	9
	Regional	598	-23	-10	-12
	All (excl. TEs)	1079	-17	-6	-9
All capacities	Intercapital	1065	-17	-7	-10
	Metro	243	-13	1	1
	Regional	324	-28	-16	-16
	All (excl. TEs)	538	-16	-4	-5
	Tail-end	.	.	.	.
	All (incl. TEs)	538	-16	-4	-5

Capacity class	*Route type*	*DAA 2012*	*Model (A4a)*	*Model (A4b)*	*Models (A4c)*
Regulated & Deregulated					
Capacity < 5 Mbps	Intercapital	38	-30	-17	-15
	Metro	16	-6	11	13
	Regional	-22	-45	-33	-32
	All (excl. TEs)	-3	-28	-14	-13
Capacity 5 - 50 Mbps	Intercapital	180	-21	-12	-12
	Metro	68	-22	-10	-9
	Regional	-3	-62	-55	-55
	All (excl. TEs)	62	-37	-28	-27
Capacity 50 - 200 Mbps	Intercapital	239	-35	-28	-29
	Metro	182	-22	-10	-10
	Regional	49	-67	-61	-62
	All (excl. TEs)	117	-49	-42	-42
Capacity 200+ Mbps	Intercapital	1211	-34	-26	-29
	Metro	605	-5	8	8
	Regional	566	-33	-22	-24
	All (excl. TEs)	825	-25	-15	-17
All capacities	Intercapital	761	-32	-24	-26
	Metro	226	-13	1	1
	Regional	137	-53	-44	-44
	All (excl. TEs)	298	-35	-25	-25
	Tail-end	160	28	55	55
	All (incl. TEs)	289	-31	-19	-19

A5 Including a Specific Effect for 2.5 Mbps Services

Table A.27: **Model A5: RE models with 2.5 Mbps dummy (2015 data excl. new obs.)**

	Model (A5a)		Model (A5b)		Model (5Ac)	
Predictor	*Coeff.*	*t-stat.*	*Coeff.*	*t-stat.*	*Coeff.*	*t-stat.*
constant	5.73409	33.82	4.80160	40.74	5.00604	55.97
log capacity	0.47222	38.14	0.46796	37.68	0.46665	37.66
log distance	0.05123	2.05	0.05093	2.03	0.06829	2.74
0.5(log capacity)^2	-0.03334	-11.45	-0.03275	-11.20	-0.03248	-11.12
0.5(log distance)^2	0.02044	3.10	0.02049	3.09	0.02053	3.08
(log capacity)(log distance)	-0.00086	-0.54	-0.00096	-0.60	-0.00187	-1.19
log route t'put	-0.01656	-3.05	-0.01825	-3.35	.	.
log ESA t'put	0.03515	3.78	0.03148	3.37	.	.
contract start date	-0.00005	-7.61	.	.	.	.
route class 2 (Metro)	0.15525	2.12	0.15953	2.16	0.20110	2.74
route class 3 (Regional)	0.31720	5.48	0.31344	5.39	0.32329	5.53
Provider #1	[redacted]					
Provider #3						
Provider #4						
Provider #5						
Provider #6						
Provider #7						
Provider #8						
Provider #9						
interface-type 3 (SDH)	0.22304	11.37	0.26208	13.78	0.25937	13.65
2 Mbps dummy	-0.14821	-5.76	-0.13936	-5.39	-0.14248	-5.51
σ(u)	0.31299		0.31507		0.32009	
σ(e)	0.42715		0.42883		0.42841	

Source: Economic Insights estimation results.

Table A.28: **Model A5: Cost elasticities**

	Model (A5a)		Model (A5b)		Model (A5c)	
	capacity	*distance*	*capacity*	*distance*	*capacity*	*distance*
Deregulated routes						
10th	0.449	0.062	0.445	0.061	0.443	0.078
50th	0.375	0.101	0.372	0.100	0.369	0.115
90th	0.276	0.180	0.275	0.180	0.269	0.192
Regulated routes						
10th	0.448	0.079	0.444	0.079	0.442	0.096
50th	0.409	0.116	0.406	0.116	0.402	0.132
90th	0.299	0.168	0.297	0.168	0.292	0.181

Source: Economic Insights.

Table A.29: **Model A5: RE models with 2.5 Mbps dummy (2015 data excl. new obs.): Goodness-of-fit**

Goodness-of-fit statistic	**Model (A5a)** *10-fold validation (avg)*	*Full sample*	**Model (A5b)** *10-fold validation (avg)*	*Full sample*	**Model (A5c)** *10-fold validation (avg)*	*Full sample*
Within-sample						
obs	6090	6767	6090	6767	6090	6767
R^2*	0.69590	0.69534	0.69412	0.69355	0.68981	0.68932
BIC		9288.2		9337.1		9334.2
RMSE (based on ue)	0.52304	0.52360	0.52448	0.52503	0.52866	0.52918
MAE (based on ue)	0.36790	0.36824	0.36851	0.36888	0.37316	0.37334
Out-of-sample						
obs	6090	0	6090	0	6090	0
R^2*	0.69411	.	0.69233	.	0.68823	.
RMSE (based on ue)	0.52462	.	0.52601	.	0.52998	.
MAE (based on ue)	0.36916	.	0.36970	.	0.37424	.

Notes: * Squared correlation between fitted and actual dependent.

Table A.30: **Model 5: RE models with 2.5 Mbps dummy (2015 data excl. new obs.): Statistical tests**

	Model (A5a) *Stat.*	*P-value**	**Model (A5b)** *Stat.*	*P-value**	**Model (A5c)** *Stat.*	*P-value**
Normality of residuals						
Doornik-Hansen(1)	4135.9	0.0000	4256.4	0.0000	4202.6	0.0000
IQR (% severe outliers) (2)†	1.62%		1.79%		1.82%	
Homoscedasticity						
Breusch-Pagan/Cook-Weisberg(6)	1352.5	0.0000	1416.3	0.0000	1471.0	0.0000
Multicollinearity						
# VIF scores > 10	5/20		5/19		5/17	
Misspecification						
RESET (7)	34.3	0.0000	20.5	0.0000	21.4	0.0000
Link test(8)	2.23	0.026	0.89	0.372	0.80	0.426
Joint significance tests						
2nd order output terms (df = 3)	154.5	0.0000	148.8	0.0000	150.4	0.0000
Route classes (df = 2)	44.5	0.0000	42.0	0.0000	38.9	0.0000
Provider fixed effects (df = 7)	1067.3	0.0000	1086.5	0.0000	1088.0	0.0000

Note: * Null hypothesis is rejected, as a standard procedure, in these tests, if P-value is less than 0.05. Equivalently, the reported statistic exceeds the critical value for that statistic; † Percentage of n = 6767 observations; (1) $chi^2(2k)$ where k = 20 for 1st model, and k = 19 for 2nd model and k = 17 for 3rd model. (2) Severe outliers represent about 0.0002% of a normal distribution; (3) Studentized residual > 3; (4) Hat value > $3k/n$; (5) Cook's D > 5 × average Cook's D; (6) $chi^2(1)$; (7) Via powers of the dependent variable, $F(3,n–k–3)$; (8) t-statistic on hat^2; (9) $F(r,n–k–r)$, where r = number of parameters tested, and r = 3 for higher-order output terms, r = 2 for route classes, and r = 8 for provider-specific effects. (10) $chibar^2(1)$; a Approximate, based on OLS regression of $(y - \hat{u}_i)$ on the predictors.

Table A.31: **Model A5: Average predicted prices by market segment ($/month)**

Capacity class	*Route type*	*Actual*	*DAA 2012*	*Model (A5a)*	*Model (A5b)*	*Models (A5c)*
Regulated routes						
Capacity < 5 Mbps	Intercapital	1209	1365	726	797	814
	Metro	375	457	366	410	425
	Regional	1043	781	569	634	663
	All (excl. TEs)	713	635	472	526	548
Capacity 5 - 50 Mbps	Intercapital	2881	6192	1604	1698	1705
	Metro	1075	1678	765	823	858
	Regional	3995	3409	1387	1508	1591
	All (excl. TEs)	2387	2809	1091	1177	1228
Capacity 50 - 200 Mbps	Intercapital	9260	15157	2752	2955	2914
	Metro	2197	6295	1633	1758	1825
	Regional	9691	13374	2890	3108	3247
	All (excl. TEs)	6388	10441	2330	2506	2599
Capacity 200+ Mbps	Intercapital	14561	61066	4686	4970	4865
	Metro	2684	16947	2494	2673	2771
	Regional	7458	48954	4677	5029	5196
	All (excl. TEs)	5611	33806	3580	3840	3951
All capacities	Intercapital	4842	12774	1830	1958	1946
	Metro	782	2321	678	740	768
	Regional	2630	5544	1157	1263	1317
	All (excl. TEs)	1811	4267	949	1034	1072
	Tail-end	[redacted]	1064	484	532	547
	All (incl. TEs)	[redacted]	3115	782	854	883
Deregulated						
Capacity < 5 Mbps	Intercapital	812	1340	686	749	762
	Metro	397	439	356	398	411
	Regional	504	653	501	559	576
	All (excl. TEs)	434	516	391	436	449
Capacity 5 - 50 Mbps	Intercapital	1534	4853	1368	1451	1457
	Metro	859	1490	723	779	800
	Regional	1901	4346	1469	1573	1659
	All (excl. TEs)	1010	2162	859	921	942
Capacity 50 - 200 Mbps	Intercapital	2474	13244	2457	2606	2590
	Metro	1680	4685	1397	1501	1540
	Regional	4432	10605	2480	2661	2767
	All (excl. TEs)	2090	6940	1708	1828	1862
Capacity 200+ Mbps	Intercapital	7744	122355	5857	6231	6060
	Metro	2514	18773	2484	2662	2730
	Regional	5421	37840	4115	4434	4556
	All (excl. TEs)	4418	52098	3684	3937	3939
All capacities	Intercapital	3316	38645	2713	2887	2841
	Metro	895	3073	798	865	889
	Regional	2116	8966	1532	1659	1715
	All (excl. TEs)	1337	8531	1128	1215	1232
	Tail-end	.	.	.	.	.
	All (incl. TEs)	1337	8531	1128	1215	1232

Capacity class	*Route type*	*Actual*	*DAA 2012*	*Model (A5a)*	*Model (A5b)*	*Models (A5c)*
Regulated & Deregulated						
Capacity < 5 Mbps	Intercapital	979	1350	703	770	784
	Metro	387	448	361	404	418
	Regional	980	766	561	625	653
	All (excl. TEs)	606	589	441	492	510
Capacity 5 - 50 Mbps	Intercapital	1842	5160	1422	1508	1514
	Metro	911	1535	733	790	814
	Regional	3675	3552	1399	1518	1602
	All (excl. TEs)	1465	2376	936	1006	1036
Capacity 50 - 200 Mbps	Intercapital	4032	13683	2525	2686	2664
	Metro	1835	5169	1468	1578	1626
	Regional	8561	12778	2802	3012	3144
	All (excl. TEs)	3857	8379	1964	2107	2165
Capacity 200+ Mbps	Intercapital	8686	113887	5695	6056	5895
	Metro	2573	18139	2488	2666	2744
	Regional	6838	45569	4506	4848	5001
	All (excl. TEs)	4874	45105	3644	3900	3944
All capacities	Intercapital	3711	31948	2484	2647	2609
	Metro	849	2764	749	814	840
	Regional	2555	6047	1212	1321	1375
	All (excl. TEs)	1584	6310	1035	1121	1149
	Tail-end	[redacted]	1064	484	532	547
	All (incl. TEs)		5124	910	988	1013

Table A.32: **Model 5: Difference in predicted prices compared to DAA 2012 (%)**

Capacity class	*Route type*	*Model (A5a)*	*Model (A5b)*	*Models (A5c)*
Regulated routes				
Capacity < 5 Mbps	Intercapital	-47	-42	-40
	Metro	-20	-10	-7
	Regional	-27	-19	-15
	All (excl. TEs)	-26	-17	-14
Capacity 5 - 50 Mbps	Intercapital	-74	-73	-72
	Metro	-54	-51	-49
	Regional	-59	-56	-53
	All (excl. TEs)	-61	-58	-56
Capacity 50 - 200 Mbps	Intercapital	-82	-81	-81
	Metro	-74	-72	-71
	Regional	-78	-77	-76
	All (excl. TEs)	-78	-76	-75
Capacity 200+ Mbps	Intercapital	-92	-92	-92
	Metro	-85	-84	-84
	Regional	-90	-90	-89
	All (excl. TEs)	-89	-89	-88
All capacities	Intercapital	-86	-85	-85
	Metro	-71	-68	-67
	Regional	-79	-77	-76
	All (excl. TEs)	-78	-76	-75
	Tail-end	-55	-50	-49

Capacity class	*Route type*	*Model (A5a)*	*Model (A5b)*	*Models (A5c)*
	All (incl. TEs)	-75	-73	-72
Deregulated				
Capacity < 5 Mbps	Intercapital	-49	-44	-43
	Metro	-19	-9	-6
	Regional	-23	-14	-12
	All (excl. TEs)	-24	-16	-13
Capacity 5 - 50 Mbps	Intercapital	-72	-70	-70
	Metro	-51	-48	-46
	Regional	-66	-64	-62
	All (excl. TEs)	-60	-57	-56
Capacity 50 - 200 Mbps	Intercapital	-81	-80	-80
	Metro	-70	-68	-67
	Regional	-77	-75	-74
	All (excl. TEs)	-75	-74	-73
Capacity 200+ Mbps	Intercapital	-95	-95	-95
	Metro	-87	-86	-85
	Regional	-89	-88	-88
	All (excl. TEs)	-93	-92	-92
All capacities	Intercapital	-93	-93	-93
	Metro	-74	-72	-71
	Regional	-83	-81	-81
	All (excl. TEs)	-87	-86	-86
	Tail-end	.	.	.
	All (incl. TEs)	-87	-86	-86
Regulated & Deregulated				
Capacity < 5 Mbps	Intercapital	-48	-43	-42
	Metro	-19	-10	-7
	Regional	-27	-18	-15
	All (excl. TEs)	-25	-17	-13
Capacity 5 - 50 Mbps	Intercapital	-72	-71	-71
	Metro	-52	-49	-47
	Regional	-61	-57	-55
	All (excl. TEs)	-61	-58	-56
Capacity 50 - 200 Mbps	Intercapital	-82	-80	-81
	Metro	-72	-69	-69
	Regional	-78	-76	-75
	All (excl. TEs)	-77	-75	-74
Capacity 200+ Mbps	Intercapital	-95	-95	-95
	Metro	-86	-85	-85
	Regional	-90	-89	-89
	All (excl. TEs)	-92	-91	-91
All capacities	Intercapital	-92	-92	-92
	Metro	-73	-71	-70
	Regional	-80	-78	-77
	All (excl. TEs)	-84	-82	-82
	Tail-end	-55	-50	-49
	All (incl. TEs)	-82	-81	-80

Table A.33: **Model A5: Difference in predicted prices compared to Actual (%)**

Capacity class	*Route type*	*DAA 2012*	*Model (A5a)*	*Model (A5b)*	*Models (A5c)*
Regulated routes					
Capacity < 5 Mbps	Intercapital	13	-40	-34	-33
	Metro	22	-2	9	13
	Regional	-25	-45	-39	-36
	All (excl. TEs)	-11	-34	-26	-23
Capacity 5 - 50 Mbps	Intercapital	115	-44	-41	-41
	Metro	56	-29	-23	-20
	Regional	-15	-65	-62	-60
	All (excl. TEs)	18	-54	-51	-49
Capacity 50 - 200 Mbps	Intercapital	64	-70	-68	-69
	Metro	186	-26	-20	-17
	Regional	38	-70	-68	-66
	All (excl. TEs)	63	-64	-61	-59
Capacity 200+ Mbps	Intercapital	319	-68	-66	-67
	Metro	531	-7	0	3
	Regional	556	-37	-33	-30
	All (excl. TEs)	502	-36	-32	-30
All capacities	Intercapital	164	-62	-60	-60
	Metro	197	-13	-5	-2
	Regional	111	-56	-52	-50
	All (excl. TEs)	136	-48	-43	-41
	Tail-end	160	18	30	34
	All (incl. TEs)	138	-40	-35	-32
Deregulated					
Capacity < 5 Mbps	Intercapital	65	-16	-8	-6
	Metro	11	-10	0	3
	Regional	29	-1	11	14
	All (excl. TEs)	19	-10	1	4
Capacity 5 - 50 Mbps	Intercapital	216	-11	-5	-5
	Metro	73	-16	-9	-7
	Regional	129	-23	-17	-13
	All (excl. TEs)	114	-15	-9	-7
Capacity 50 - 200 Mbps	Intercapital	435	-1	5	5
	Metro	179	-17	-11	-8
	Regional	139	-44	-40	-38
	All (excl. TEs)	232	-18	-13	-11
Capacity 200+ Mbps	Intercapital	1480	-24	-20	-22
	Metro	647	-1	6	9
	Regional	598	-24	-18	-16
	All (excl. TEs)	1079	-17	-11	-11
All capacities	Intercapital	1065	-18	-13	-14
	Metro	243	-11	-3	-1
	Regional	324	-28	-22	-19
	All (excl. TEs)	538	-16	-9	-8
	Tail-end	.	.	.	.
	All (incl. TEs)	538	-16	-9	-8

Capacity class	*Route type*	*DAA 2012*	*Model (A5a)*	*Model (A5b)*	*Models (A5c)*
Regulated & Deregulated					
Capacity < 5 Mbps	Intercapital	38	-28	-21	-20
	Metro	16	-7	5	8
	Regional	-22	-43	-36	-33
	All (excl. TEs)	-3	-27	-19	-16
Capacity 5 - 50 Mbps	Intercapital	180	-23	-18	-18
	Metro	68	-20	-13	-11
	Regional	-3	-62	-59	-56
	All (excl. TEs)	62	-36	-31	-29
Capacity 50 - 200 Mbps	Intercapital	239	-37	-33	-34
	Metro	182	-20	-14	-11
	Regional	49	-67	-65	-63
	All (excl. TEs)	117	-49	-45	-44
Capacity 200+ Mbps	Intercapital	1211	-34	-30	-32
	Metro	605	-3	4	7
	Regional	566	-34	-29	-27
	All (excl. TEs)	825	-25	-20	-19
All capacities	Intercapital	761	-33	-29	-30
	Metro	226	-12	-4	-1
	Regional	137	-53	-48	-46
	All (excl. TEs)	298	-35	-29	-27
	Tail-end	160	18	30	34
	All (incl. TEs)	289	-31	-25	-23

A6 Piecewise Regression

Table A.34: **Model A6: Piecewise RE models (2015 data excl. new obs.)***

	Model (A6a)		Model (A6b)		Model (A6c)	
Predictor	*Coeff.*	*t-stat.*	*Coeff.*	*t-stat.*	*Coeff.*	*t-stat.*
constant	-1.59012	-0.32	2.98840	0.60	2.75944	0.56
log capacity	18.35829	1.46	4.48298	0.36	5.57840	0.45
log distance	0.25402	6.36	0.23972	5.98	0.26165	6.59
0.5(log capacity)^2	-21.68648	-1.38	-4.64703	-0.30	-5.98830	-0.38
0.5(log distance)^2	0.01441	2.22	0.01465	2.24	0.01460	2.22
(log capacity)(log distance)	-0.20670	-4.85	-0.19176	-4.48	-0.19715	-4.62
Δlog capacity**	-17.86012	-1.42	-3.97968	-0.32	-5.07644	-0.41
Δ0.5(log capacity)^2**	21.64656	1.38	4.60613	0.30	5.94764	0.38
Δ(log capacity)(log distance)**	0.20871	4.77	0.19350	4.40	0.19805	4.51
log route t'put	-0.01651	-3.03	-0.01829	-3.34	.	.
log ESA t'put	0.03566	3.82	0.03234	3.45	.	.
contract start date	-0.00005	-7.66	.	.	.	.
route class 2 (Metro)	0.16847	2.29	0.16999	2.30	0.21273	2.88
route class 3 (Regional)	0.31406	5.40	0.30813	5.27	0.31800	5.41
Provider #1						
Provider #3						
Provider #4						
Provider #5						
Provider #6						
Provider #7						
Provider #8						
Provider #9						
interface-type 3 (SDH)	0.19287	8.75	0.23181	10.76	0.22858	10.61
σ(u)	0.31559		0.31776		0.32295	
σ(e)	0.42696		0.42866		0.42821	

Source: Economic Insights estimation results.

Notes: * Known knot at 2.5 Mbps. ** Additive effect for the increment above the knot.

Table A.35: **Model A6: Cost elasticities**

	Model (A6a)		Model (A6b)		Model (A6c)	
	capacity	*distance*	*capacity*	*distance*	*capacity*	*distance*
Deregulated routes						
10th	3.214	0.119	1.158	0.115	1.321	0.133
50th	0.389	0.296	0.391	0.282	0.388	0.301
90th	0.284	0.360	0.281	0.346	0.276	0.363
Regulated routes						
10th	3.038	0.131	0.994	0.127	1.152	0.145
50th	0.433	0.305	0.435	0.291	0.431	0.311
90th	0.309	0.350	0.307	0.336	0.302	0.353

Source: Economic Insights.

Table A.36: **Model A6: Goodness-of-fit**

Goodness-of-fit statistic	**Model (A6a)** *10-fold validation (avg)*	*Full sample*	**Model (A6b)** *10-fold validation (avg)*	*Full sample*	**Model (A6c)** *10-fold validation (avg)*	*Full sample*
Within-sample						
obs	6090	6767	6090	6767	6090	6767
R^2*	0.69460	0.69401	0.69261	0.69201	0.68801	0.68752
BIC		9314.4		9364.0		9361.4
RMSE (based on ue)	0.52413	0.52471	0.52574	0.52632	0.53014	0.53067
MAE (based on ue)	0.36933	0.36968	0.37000	0.37038	0.37495	0.37515
Out-of-sample						
obs	677	6.1.1.	677	0	677	0
R^2*	0.69255	.	0.69054	.	0.68617	.
RMSE (based on ue)	0.52593	.	0.52751	.	0.53168	.
MAE (based on ue)	0.37070	.	0.37127	.	0.37617	.

Notes: * Squared correlation between fitted and actual dependent.

Table A.37: **Model A6: Statistical tests**

	Model (A6a) *Stat.*	*P-value**	**Model (A6b)** *Stat.*	*P-value**	**Model (A6c)** *Stat.*	*P-value**
Normality of residuals						
Doornik-Hansen(1)	4103.7	0.0000	4204.9	0.0000	4154.2	0.0000
IQR (% severe outliers) (2)†	1.76%		2.04%		2.08%	
Homoscedasticity						
Breusch-Pagan/Cook-Weisberg(6)	1344.6	0.0000	1402.3	0.0000	1456.7	0.0000
Multicollinearity						
# VIF scores > 10	9/22		9/21		9/19	
Misspecification						
RESET (7)	31.4	0.0000	19.2	0.0000	20.2	0.0000
Link test(8)	2.20	0.028	0.88	0.382	0.77	0.444
Joint significance tests						
2nd order output terms (df = 3)	29.1	0.0000	24.1	0.0000	25.2	0.0000
Incremental effects (df = 3)**	24.5	0.0000	19.8	0.0002	20.7	0.0001
Route classes (df = 2)	40.8	0.0000	38.2	0.0000	35.4	0.0000
Provider fixed effects (df = 7)	1026.8	0.0000	1049.6	0.0000	1051.1	0.0000

Note: * Null hypothesis is rejected, as a standard procedure, in these tests, if P-value is less than 0.05. Equivalently, the reported statistic exceeds the critical value for that statistic; ** Additive effects for incremental capacity above the knot. † Percentage of n = 6767 observations; (1) $chi^2(2k)$ where k = 22 for 1st model, and k = 21 for 2nd model and k = 19 for 3rd model. (2) Severe outliers represent about 0.0002% of a normal distribution; (3) Studentized residual > 3; (4) Hat value > $3k/n$; (5) Cook's D > 5 × average Cook's D; (6) $chi^2(1)$; (7) Via powers of the dependent variable, F(3,n–k–3); (8) t-statistic on hat^2; (9) F(r,n–k–r), where r = number of parameters tested, and r = 3 for higher-order output terms, r = 2 for route classes, and r = 8 for provider-specific effects. (10) $chibar^2(1)$; a Approximate, based on OLS regression of $(y - \hat{u}_i)$ on the predictors.

Table A.38: **Model A6: Predicted prices by market segment ($/month)**

Capacity class	*Route type*	*Actual*	*DAA 2012*	*Model (A6a)*	*Model (A6b)*	*Models (A6c)*
Regulated routes						
Capacity < 5 Mbps	Intercapital	1209	1365	787	848	868
	Metro	375	457	364	409	424
	Regional	1043	781	583	650	682
	All (excl. TEs)	713	635	479	534	557
Capacity 5 - 50 Mbps	Intercapital	2881	6192	1568	1655	1662
	Metro	1075	1678	778	828	865
	Regional	3995	3409	1347	1452	1534
	All (excl. TEs)	2387	2809	1079	1154	1205
Capacity 50 - 200 Mbps	Intercapital	9260	15157	2758	2960	2920
	Metro	2197	6295	1687	1808	1882
	Regional	9691	13374	2926	3135	3282
	All (excl. TEs)	6388	10441	2372	2542	2641
Capacity 200+ Mbps	Intercapital	14561	61066	4769	5042	4940
	Metro	2684	16947	2572	2745	2851
	Regional	7458	48954	4733	5055	5236
	All (excl. TEs)	5611	33806	3649	3893	4014
All capacities	Intercapital	4842	12774	1859	1980	1970
	Metro	782	2321	688	750	779
	Regional	2630	5544	1172	1273	1331
	All (excl. TEs)	1811	4267	962	1045	1086
	Tail-end	[redacted]	1064	443	489	502
	All (incl. TEs)		3115	776	845	876
Deregulated						
Capacity < 5 Mbps	Intercapital	812	1340	739	796	812
	Metro	397	439	356	397	411
	Regional	504	653	507	565	583
	All (excl. TEs)	434	516	395	439	453
Capacity 5 - 50 Mbps	Intercapital	1534	4853	1331	1405	1411
	Metro	859	1490	729	778	800
	Regional	1901	4346	1454	1545	1632
	All (excl. TEs)	1010	2162	857	911	933
Capacity 50 - 200 Mbps	Intercapital	2474	13244	2464	2611	2596
	Metro	1680	4685	1428	1528	1571
	Regional	4432	10605	2518	2690	2804
	All (excl. TEs)	2090	6940	1735	1851	1889
Capacity 200+ Mbps	Intercapital	7744	122355	5912	6238	6075
	Metro	2514	18773	2508	2677	2750
	Regional	5421	37840	4177	4474	4609
	All (excl. TEs)	4418	52098	3722	3953	3961
All capacities	Intercapital	3316	38645	2730	2887	2843
	Metro	895	3073	806	870	895
	Regional	2116	8966	1551	1671	1732
	All (excl. TEs)	1337	8531	1138	1220	1238
	Tail-end	.	.	.	.	.
	All (incl. TEs)	1337	8531	1138	1220	1238
Regulated & Deregulated						

Capacity class	*Route type*	*Actual*	*DAA 2012*	*Model (A6a)*	*Model (A6b)*	*Models (A6c)*
Capacity < 5 Mbps	Intercapital	979	1350	759	818	835
	Metro	387	448	360	403	417
	Regional	980	766	574	640	670
	All (excl. TEs)	606	589	447	498	518
Capacity 5 - 50 Mbps	Intercapital	1842	5160	1385	1462	1468
	Metro	911	1535	741	790	816
	Regional	3675	3552	1363	1466	1549
	All (excl. TEs)	1465	2376	930	991	1023
Capacity 50 - 200 Mbps	Intercapital	4032	13683	2532	2691	2670
	Metro	1835	5169	1506	1612	1664
	Regional	8561	12778	2838	3039	3179
	All (excl. TEs)	3857	8379	1997	2135	2198
Capacity 200+ Mbps	Intercapital	8686	113887	5754	6073	5918
	Metro	2573	18139	2530	2701	2785
	Regional	6838	45569	4564	4878	5045
	All (excl. TEs)	4874	45105	3694	3930	3981
All capacities	Intercapital	3711	31948	2505	2652	2617
	Metro	849	2764	758	821	848
	Regional	2555	6047	1227	1332	1390
	All (excl. TEs)	1584	6310	1047	1129	1159
	Tail-end	[redacted]	1064	443	489	502
	All (incl. TEs)		5124	910	984	1010

Table A.39: **Model A6: Difference in predicted prices compared to DAA 2012 (%)**

Capacity class	*Route type*	*Model (A6a)*	*Model (A6b)*	*Models (A6c)*
Regulated routes				
Capacity < 5 Mbps	Intercapital	-42	-38	-36
	Metro	-20	-11	-7
	Regional	-25	-17	-13
	All (excl. TEs)	-25	-16	-12
Capacity 5 - 50 Mbps	Intercapital	-75	-73	-73
	Metro	-54	-51	-48
	Regional	-60	-57	-55
	All (excl. TEs)	-62	-59	-57
Capacity 50 - 200 Mbps	Intercapital	-82	-80	-81
	Metro	-73	-71	-70
	Regional	-78	-77	-75
	All (excl. TEs)	-77	-76	-75
Capacity 200+ Mbps	Intercapital	-92	-92	-92
	Metro	-85	-84	-83
	Regional	-90	-90	-89
	All (excl. TEs)	-89	-88	-88
All capacities	Intercapital	-85	-85	-85
	Metro	-70	-68	-66
	Regional	-79	-77	-76
	All (excl. TEs)	-77	-76	-75
	Tail-end	-58	-54	-53

Capacity class	*Route type*	*Model (A6a)*	*Model (A6b)*	*Models (A6c)*
	All (incl. TEs)	-75	-73	-72
Deregulated				
Capacity < 5 Mbps	Intercapital	-45	-41	-39
	Metro	-19	-10	-6
	Regional	-22	-13	-11
	All (excl. TEs)	-24	-15	-12
Capacity 5 - 50 Mbps	Intercapital	-73	-71	-71
	Metro	-51	-48	-46
	Regional	-67	-64	-62
	All (excl. TEs)	-60	-58	-57
Capacity 50 - 200 Mbps	Intercapital	-81	-80	-80
	Metro	-70	-67	-66
	Regional	-76	-75	-74
	All (excl. TEs)	-75	-73	-73
Capacity 200+ Mbps	Intercapital	-95	-95	-95
	Metro	-87	-86	-85
	Regional	-89	-88	-88
	All (excl. TEs)	-93	-92	-92
All capacities	Intercapital	-93	-93	-93
	Metro	-74	-72	-71
	Regional	-83	-81	-81
	All (excl. TEs)	-87	-86	-85
	Tail-end	.	.	.
	All (incl. TEs)	-87	-86	-85
Regulated & Deregulated				
Capacity < 5 Mbps	Intercapital	-44	-39	-38
	Metro	-20	-10	-7
	Regional	-25	-16	-13
	All (excl. TEs)	-24	-16	-12
Capacity 5 - 50 Mbps	Intercapital	-73	-72	-72
	Metro	-52	-49	-47
	Regional	-62	-59	-56
	All (excl. TEs)	-61	-58	-57
Capacity 50 - 200 Mbps	Intercapital	-81	-80	-80
	Metro	-71	-69	-68
	Regional	-78	-76	-75
	All (excl. TEs)	-76	-75	-74
Capacity 200+ Mbps	Intercapital	-95	-95	-95
	Metro	-86	-85	-85
	Regional	-90	-89	-89
	All (excl. TEs)	-92	-91	-91
All capacities	Intercapital	-92	-92	-92
	Metro	-73	-70	-69
	Regional	-80	-78	-77
	All (excl. TEs)	-83	-82	-82
	Tail-end	-58	-54	-53
	All (incl. TEs)	-82	-81	-80

Table A.40: **Model A6: Difference in predicted prices compared to Actual (%)**

Capacity class	*Route type*	*DAA 2012*	*Model (6a)*	*Model (6b)*	*Models (6c)*
Regulated routes					
Capacity < 5 Mbps	Intercapital	13	-35	-30	-28
	Metro	22	-3	9	13
	Regional	-25	-44	-38	-35
	All (excl. TEs)	-11	-33	-25	-22
Capacity 5 - 50 Mbps	Intercapital	115	-46	-43	-42
	Metro	56	-28	-23	-19
	Regional	-15	-66	-64	-62
	All (excl. TEs)	18	-55	-52	-50
Capacity 50 - 200 Mbps	Intercapital	64	-70	-68	-68
	Metro	186	-23	-18	-14
	Regional	38	-70	-68	-66
	All (excl. TEs)	63	-63	-60	-59
Capacity 200+ Mbps	Intercapital	319	-67	-65	-66
	Metro	531	-4	2	6
	Regional	556	-37	-32	-30
	All (excl. TEs)	502	-35	-31	-28
All capacities	Intercapital	164	-62	-59	-59
	Metro	197	-12	-4	0
	Regional	111	-55	-52	-49
	All (excl. TEs)	136	-47	-42	-40
	Tail-end	160	8	20	23
	All (incl. TEs)	138	-41	-35	-33
Deregulated					
Capacity < 5 Mbps	Intercapital	65	-9	-2	0
	Metro	11	-10	0	3
	Regional	29	1	12	16
	All (excl. TEs)	19	-9	1	4
Capacity 5 - 50 Mbps	Intercapital	216	-13	-8	-8
	Metro	73	-15	-9	-7
	Regional	129	-24	-19	-14
	All (excl. TEs)	114	-15	-10	-8
Capacity 50 - 200 Mbps	Intercapital	435	0	6	5
	Metro	179	-15	-9	-6
	Regional	139	-43	-39	-37
	All (excl. TEs)	232	-17	-11	-10
Capacity 200+ Mbps	Intercapital	1480	-24	-19	-22
	Metro	647	0	6	9
	Regional	598	-23	-17	-15
	All (excl. TEs)	1079	-16	-11	-10
All capacities	Intercapital	1065	-18	-13	-14
	Metro	243	-10	-3	0
	Regional	324	-27	-21	-18
	All (excl. TEs)	538	-15	-9	-7
	Tail-end	.	.	.	.
	All (incl. TEs)	538	-15	-9	-7

Capacity class	*Route type*	*DAA 2012*	*Model (6a)*	*Model (6b)*	*Models (6c)*
Regulated & Deregulated					
Capacity < 5 Mbps	Intercapital	38	-22	-16	-15
	Metro	16	-7	4	8
	Regional	-22	-41	-35	-32
	All (excl. TEs)	-3	-26	-18	-15
Capacity 5 - 50 Mbps	Intercapital	180	-25	-21	-20
	Metro	68	-19	-13	-10
	Regional	-3	-63	-60	-58
	All (excl. TEs)	62	-37	-32	-30
Capacity 50 - 200 Mbps	Intercapital	239	-37	-33	-34
	Metro	182	-18	-12	-9
	Regional	49	-67	-64	-63
	All (excl. TEs)	117	-48	-45	-43
Capacity 200+ Mbps	Intercapital	1211	-34	-30	-32
	Metro	605	-2	5	8
	Regional	566	-33	-29	-26
	All (excl. TEs)	825	-24	-19	-18
All capacities	Intercapital	761	-33	-29	-29
	Metro	226	-11	-3	0
	Regional	137	-52	-48	-46
	All (excl. TEs)	298	-34	-29	-27
	Tail-end	160	8	20	23
	All (incl. TEs)	289	-31	-25	-23